## Differential Geometry of Curves and Surfaces (MTH-406)

 Quiz (04/02/2017)Time: 60 minutes Maximum Marks: 10

## Attempt all questions. Use separate page for each answer.

## Problem 1.

(1) Show that an ellipse has exactly four vertices.
(2) Let $\sigma: \mathbb{R}^{2} \rightarrow \mathbb{R}^{3}$ be given by

$$
\sigma(u, v)=\left(u, v, u^{2}-v^{2}\right) .
$$

Prove that this is a smooth surface patch and compute the equation of the tangent plane at $(1,1,0)$.

Problem 2. Suppose that two smooth surfaces $S_{1}$ and $S_{2}$ are diffeomorphic and that $S_{1}$ is orientable. Prove that $S_{2}$ is orientable.

$$
\begin{equation*}
O R \tag{4}
\end{equation*}
$$

State and prove the Inverse Function Theorem for smooth(regular) surfaces.
Hint: Use the Inverse Function theorem for $\mathbb{R}^{2}$,

