DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES (MTH-406)

QUIZ (04/02/2017)

Time: 60 minutes Maximum Marks: 10

Attempt all questions. Use separate page for each answer.

Problem 1.

- (1) Show that an ellipse has exactly four vertices.
- (2) Let $\sigma : \mathbb{R}^2 \to \mathbb{R}^3$ be given by

$$\sigma(u, v) = (u, v, u^2 - v^2)$$

Prove that this is a smooth surface patch and compute the equation of the tangent plane at (1, 1, 0). (3)

Problem 2. Suppose that two smooth surfaces S_1 and S_2 are diffeomorphic and that S_1 is orientable. Prove that S_2 is orientable.

OR

State and prove the Inverse Function Theorem for smooth(regular) surfaces. (4)

Hint: Use the Inverse Function theorem for \mathbb{R}^2 ,

(3)