Differential Geometry of Curves and Surfaces (MTH-406)

Mid semester examination (06/03/2017)

Time: 120 minutes Maximum Marks: 25

Solve all questions.

Problem 1. Answer the following questions "True" or "False".

(1) Every local isometry is an equiareal map.
(2) Let T be a spherical triangle on the unit sphere S^2 with internal angles α, β and γ, then $\alpha + \beta + \gamma < \pi$.
(3) Torsion of a plane curve is zero.
(4) A curve has a unit speed reparametrization if it is regular. (4)

Problem 2. Show that Enneper's surface

$$\sigma(u, v) = (u - u^3/3 + uv^2, v - v^3/3 + vu^2, u^2 - v^2)$$

is conformally parametrized. (4)

Problem 3. Show that geodesics in a plane are straight lines and vice versa. (3)

Problem 4. Let γ be a unit-speed regular curve on a oriented surface. Prove that the normal curvatures of γ are (4)

$$\kappa_n =<< d\gamma/dt, d\gamma/dt >> .$$

Problem 5. Let $S = \{(x, y, z) \in \mathbb{R}^3 | x^2 + y^2 + z^2 = 1$ be an unit sphere. Find the geodesic curvature of the circle γ in the unit sphere S, which is the intersection of the sphere with the plane $z = c$, where c is a constant with $-1 < c < 1$. Compare it with the curvature of the same circle considered as a curve in the space. (4)

Problem 6. Let S be a surface (Helicoid) parametrized by

$$\sigma(t, v) = (vcost, vsint, \alpha t),$$

where α is a constant real number. Compute the Gauss, Mean and principal curvature. Find all elliptic parabolic and hyperbolic points on the surface? (6)