DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES (MTH-406)

END SEMESTER EXAMINATION (18/04/2017)

Time: 180 minutes Maximum Marks: 50

Solve all questions.

Problem 1. Answer the following questions "True" or "False".

(1) The curve $\gamma(t) = (e^t, t^2)$ for $t \in \mathbb{R}$ is a regular curve.
(2) There is a unique geodesic passing through any two distinct points of a sphere.
(3) There exist a compact surface, whose all points are hyperelliptic.
(4) There is a unique geodesic through any given point of a surface in any given tangent direction. (4)

Problem 2.

(1) Compute the curvature of the curve: (4)

$$\gamma(t) = (cos^3t, sin^3t).$$

(2) Show that the signed curvature of any regular plane curve $\gamma(t)$ is a smooth function of t. (4)
(3) Let $f : S_1 \to S_2$ be a local diffeomorphism and let γ be a regular curve on S_1. Show that $f \circ \gamma$ is a regular curve on S_2. (4)
(4) Let γ be a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Then show that, γ is a parametrization of (part of) a circle. (5)
(5) What is the effect on the first and second fundamental form of a surface of applying an isometry of $\mathbb{R}^3$? Or a dilation (i.e., a map $\mathbb{R}^3 \to \mathbb{R}^3$ of the form $v \to \lambda v$ for some constant $\lambda \neq 0$)? (5)
(6) Find the geodesics on the unit cylinder. Show that, if p and q are distinct points of the unit cylinder, there are either two or infinitely many geodesics on the cylinder with endpoints p and q (and which do not otherwise pass through p or q). Which pairs p, q have the former property? (6)
(7) State the following theorem (Write all assumptions and notations clearly).
   (a) Gauss Theorema Egregium.
   (b) Gauss-Bonnet for simple closed curves.
   (c) Gauss-Bonnet for compact surfaces. (1+2+2)
(8) Define minimal surface. Show that the parametrization of the catenoid

$$\sigma(u, v) = (coshu\ cosv, coshu\ sinv, u).$$

is conformally parametrized. Using the above parametrization of the catenoid show that it is a minimal surface. (6)
(9) Show that a map that is both conformal and equiareal is a local isometry. Give an example of an equiareal map that is not a local isometry. (5+2)