DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES (MTH 406)

Mid Semester Examination (23/02/2020)

Time: 120 minutes Maximum Marks: 30

Attempt all questions. Use separate page for each answer.

Problem-1: Are the following statements **"True"** or **"False"**? DO NOT write any justification or a proof. (4)

(a). Any unit speed curve in $\mathbb{R}^3$ with constant curvature is a part of circle.
(b). The normal curvature of a unit speed curve on a smooth surface depends only on the second fundamental form of the surface.
(c). The ellipse $\gamma(t) = (3\cos(t), 5\sin(t))$ has at least three vertex.
(d). Let S be a surface and P be a point on the surface. Let $\mathcal{S}$ be the set of all curves on the surface which passes through P and obtained by the intersection of S with the plane which is normal to the tangent plane at P and passing through P. The normal curvature of any curve in $\mathcal{S}$ is same at P..

Problem-2: Let (a_{ij}) be a skew-symetric 3×3 matrix. Let X_1, X_2 and X_3 be smooth functions of a parameter s satisfying the differentail equations

$$\dot{X}_i = \sum_{j=1}^{3} a_{ij} X_j,$$

for $i = 1, 2$ and 3, and suppose that for some parameter value s_0 the vectors $X_1(s_0), X_2(s_0)$ and $X_3(s_0)$ are orthonormal. Show that the vectors $X_1(s), X_2(s)$ and $X_3(s)$ are orthonormal for all values of s. (6)

Problem-3: Show that applying an isometry of $\mathbb{R}^3$ to a surface does not change its first fundamental form. What is the effect of a dialation (i.e., a map $\mathbb{R}^3 \to \mathbb{R}^3$ of the form $v \to av$ for some constant real number $a \neq 0$)? (6)

Problem-4: Give a definition of orientable surface.
Suppose that two smooth surfaces S_1 and S_2 are diffeomorphic and that S_1 is orientable. Prove that S_2 is also orientable. (1+6)

Problem-5: Let

$$S = \{(x; y; z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\}$$

be an unit sphere. Find the geodesic curvature of the circle in the unit sphere S, which is the intersection of the sphere with the plane $z = c$, where c is a constant real number and $-1 < c < 1$. Compare it with the curvature of the same circle considered as a curve in the space. (7)