

**DIFFERENTIAL GEOMETRY OF CURVES AND SURFACES**  
(MTH 406)  
**ASSIGMENT-2**

SUBMISSION DATE: 21-01-2020

**Problem-A.** Submit an answer of all problems.

- (1) Show that, if the curvature  $\kappa(t)$  of a regular curve  $\gamma(t)$  is  $> 0$  everywhere, then  $\kappa(t)$  is a smooth function of  $t$ . Give an example to show that this may not be the case without the assumption that  $\kappa > 0$ .
- (2) Let  $\gamma$  be a regular curve in  $\mathbb{R}^3$  with nowhere vanishing curvature. Show that, the image of  $\gamma$  is contained in a plane if and only if torsion  $\tau$  is zero at every point of the curve.
- (3) Solve exercise 2.1.1(i), 2.1.1(iii), 2.3.1, 2.3.2 from Chapter 2.
- (4) Find an unit speed curve plane curve whose curvature function is  $1/(1+s^2)$ .

**Problem:B.** You don't need to submit it.

- (1) Let  $\gamma$  be a unit-speed curve in  $\mathbb{R}^3$  with constant curvature and zero torsion. Show that  $\gamma$  is (part of) a circle.
- (2) Solve 2.2.1, 2.2.2, 2.2.4, 2.3.4, 2.3.5, 2.3.6 from Chapter 2.
- (3) Find an unit speed curve space curve whose curvature and torsion function are  $1/(1+s^2)$ .
- (4) Find the curvature function for the curve  $\gamma(t) = (t, \sin(t))$ .

**Text Book:** Elementry Differential Geometry, Andrew Pressley 2nd edition