Fields and Galois Theory (MTH 401)

Quiz-2 (27/10/2020)

Time: 50 minutes Maximum Marks: 23

Notation:

(1) $Aut(E/F) = \{\sigma : E \to E : \sigma$ is an automorphism of E and $\sigma|_F = Id_F\}$.
(2) $\mathbb{F}_p$ denotes the field of order p.
(3) $\mathbb{R}$ = field of real numbers, $\mathbb{Q}$ = field of rational numbers .

Write down your Name and Roll number on the front page. Attempt all questions.

Problem-A:. Are the following claims **true** or **false**?

For the correct answer you will get 1 mark and for the wrong answer you will get -1 mark. (7)

(1) Let $k \subset E$ be fields and $\alpha \in E$. Then $k[\alpha] = k(\alpha)$.

(2) We can find fields $k \subset E \subset L$ such that $1 < [L : E] < \infty$ such that every element of L, which is algebraic over k, is in k.

(3) Let E be a finite algebraic extension over a finite field k. The number of fields F such that $k \subset F \subset E$ may be infinite.

(4) The cardinality of the set $Aut(\mathbb{Q}(\sqrt[5]{3})/\mathbb{Q})$ is 5.

(5) If E/F is a finite extension of fields then F is perfect iff E is perfect.

(6) $\mathbb{R}$ has a field extension of odd degree > 1.

(7) Let E be an algebraic extension over k then every polynomial in $E[x]$ splits over k iff every polynomial in $k[x]$ splits over k.

Problem-B:. For the correct answer you will get 2 mark and for the wrong answer you will get -1 mark. (8)

(1) Let p be a prime and F_p be the field with p elements. For which prime p is the polynomial $z^2 + z + 1$ over F_p irreducible ?
A) 11
B) 13
C) 7
D) 31

(2) The number of all degree 3 irreducible polynomials over $\mathbb{F}_{11}$ is
A) 2520
B) 4400
C) 55
D) 11

(3) The degree of $\mathbb{Q}(\sqrt{2}, \sqrt{6}, \sqrt{15})$ over $\mathbb{Q}$ is.
A) 2
B) 4
C) 9
D) 8

(4) If E be the splitting field of the polynomial $x^3 - 2$ over $\mathbb{Q}$. Then choose which statements is true.
A) $\sqrt{-1} \in E$.

B) $[E : \mathbb{Q}] = 3$

C) $\frac{E[x]}{<x^2 - \sqrt[3]{2}>} \cong \frac{E[x]}{<2x^2 - \sqrt[3]{2}(\sqrt{-3}-1)>}$

D) $\frac{E[x]}{<x^2 - \sqrt[3]{2}>} \cong \frac{E[x]}{<2x^2 - \sqrt[3]{3}>}$

Problem-C:. Choose all correct answers. Multiple answer can be true. **There is no minus marking.** (8)

(1) Let E be an algebraic extension over k. Then choose which statements are true.
- A) If E is separable extension over k then E is normal extension over k.
- B) If E is splitting field of a polynomial over k then E is normal extension.
- C) If E is splitting field of a polynomial over k and char $k = 0$ then E is normal and separable over k.
- D) If E is separable extension over k and for all $\alpha \in E, deg(irr(\alpha, k) \leq n$ then $[E : k] \leq n$.

(2) Let E/k be a finite extension of fields. Which of the following assertions are correct:
- A) If $E = k(\alpha)$, where α is a root of a separable polynomial in $k[X]$, then E/k is separable.
- B) There exists $\alpha \in E$ such that $E = k(\alpha)$.
- C) For any embedding σ of k in an algebraically closed field L, there exists $\tau : E \to L$ which extends σ.
- D) For any embedding σ of k in an algebraically closed field L, there exists infinitely many embedding $\tau : E \to L$ which extends σ.

(3) Let E be an algebraic extension over a finite field k. Which of the following assertions are correct:
- A) E is separable over k.
- B) E is normal over k.
- C) If E is finite extension over k then E is normal extension.
- D) If the cardinality of $Aut(E/k)$ is n then $[E : k] = n$.

(4) Which of the following statements are correct:
- A) $\frac{\mathbb{Q}[x]}{<x^2-5>} \cong \mathbb{Q}(\sqrt{7})$
- B) $\frac{\mathbb{F}_3[x]}{<x^3-x+1>} \cong \frac{\mathbb{F}_3[x]}{<x^3-x-1>}$.
- C) $\frac{\mathbb{Q}[x]}{<x^2+x+1>} \cong \frac{\mathbb{Q}[x]}{<x^2+5x+7>}$.
- D) $\frac{\mathbb{Q}[x]}{<x^2-2>} \cong \frac{\mathbb{Q}[x]}{<x^2-3>}$