Fields and Galois Theory (MTH 401)

Quiz-1 (12/09/2020)

Time: 60 minutes Maximum Marks: 23

Attempt all questions.

Problem-A:. Are the following claims **true** or **false**?

Do not justfy your answer. For the correct answer you wll get 1 mark and for the wrong answer you will get -1 marks. (7)

(1) Every field has non-trivial field extensions.
(2) Every field has non-trivial algebraic field extensions.
(3) Two Extensions of the same degree of a field are isomorphic.
(4) Every algebraic extension of a field is finite extension.
(5) Every algebraic extension of $\mathbb{Q}$ is finite extension (i.e finite dimesion vector space over $\mathbb{Q}$.
(6) Every extension of a finite field is a finite extension.
(7) The polynomial $x^3 + 3x - 2\pi$ is irreducible over $\mathbb{R}$.

Problem-B:. Wrtie down all $\mathbb{Q}$-isomorphism between (5)

- $\mathbb{Q}(\sqrt{5})$ to $\mathbb{Q}(\sqrt{7})$.
- $\mathbb{Q}(\sqrt{5}, \sqrt{7})$ to $\mathbb{Q}(\sqrt{5}, \sqrt{7})$.

Problem-C:. Let F be a finite extension of K such that $[F : K] = p$, a prime number. If $u \in F$ but $u \notin K$, show that $F = K(u)$. (3)

Problem-D:. Let $f(x)$ be an irreducible polynomial in $K[x]$. Show that if F is an extension field of K such that $deg(f(x))$ is relatively prime to $[F : K]$, then $f(x)$ is irreducible in $F[x]$. (5)

Problem-E:. Consider $3x^2 + 4x + 3 \in F_5[x]$. Show it factors both as $(3x + 2)(x + 4)$ and as $(4x + 1)(2x + 3)$. Explain why this does NOT contradict unique factorization of polynomials. Here $F_5 = \mathbb{Z}/5\mathbb{Z}$. (3)