Fields and Galois Theory (MTH 401)

End Sememster Examination (11/12/2020)

Time: 55 minutes   Maximum Marks: 20

Notation:

(1) $Aut(E/F) = \{\sigma : E \to E : \sigma$ is an automorphism of E and $\sigma|_F = Id_F\}$.
(2) $\mathbb{F}_q$ denotes the field of order q, where $q = p^n$ for some prime p.
(3) $\bar{F}$ denotes the algebraic closure of any field F.

Write down your Name and Roll number on the front page. Attempt all questions. Each steps should be very clear. If you are using any fact then mention it very clearly.

Problem-A:. Are the following claims **True** or **False**? Do not give any justification.

For the correct answer you will get 2 mark and for the wrong answer you will get -1 mark. (10)

(1) The number of elements in $Aut(\mathbb{R}/\mathbb{Q}(\sqrt{2})$ is more then 1.

(2) The Galois group of the polynomial $(x^3 - 3)(x^3 - 2) \in \mathbb{Q}[x]$ over $\mathbb{Q}$ is abelian.

(3) Let E be a Galois extension over k. If E/k is a finite abelian extension then every intermediate field is an abelian extension of k.

(4) Let E/k be a finite Galois extension and α lie in some extension of E with minimal polynomial $f(X)$ in $E[x]$. The minimal polynomial of α in $k[X]$ is the product of all the different values of $(\sigma f)(x)$ as $\sigma \in Gal(E/k)$.

(5) Let F_1 and F_2 be finite subfields of a field K. If F_1 and F_2 have the same number of elements, then $F_1 = F_2$.

Problem-B:. Show that every element of $\mathrm{Aut}(\bar{\mathbb{F}}_q/\mathbb{F}_q)$, except for the identity map of $\bar{\mathbb{F}}_q$, has infinite order. Write each step very clearly. (10)