FIELDS AND GALOIS THEORY

MTH 401, SEMESTER 1, 2020-2021

Course Information

- **Instructor:** Dr. Sanjay Kumar Singh <sanjayks@iiserb.ac.in>
- **Office:** 210, Academic Building 1.
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- **Webpage:** http://home.iiserb.ac.in/∼sanjayks.
- **Leture Time table:** 3 PM to 3.50 PM, Tue, Thu.
- **Leture Venue:** Google Meet or Microsoft team.
- **Office Hour:** Wednesday $5.00 - 6.00$ PM. If you cannot come during my office hours please send me an email to make an appointment.

Learning Objectives:

Field Extensions are studied in an attempt to find a formula for the roots of polynomial equations, similar to the one that exists for a quadratic equation. The Galois group is introduced as a way to capture the symmetry between these roots; and the solvability of the Galois group determines if such a formula exists or not. In the 19th century, Galois proved that a formula does not exist for a general 5th degree equation. More importantly, the use of groups to study the symmetry of other objects is a pervasive theme in Mathematics, and this is traditionally the first place where one encounters it. The topics to be covered include irreducibility of polynomials, Field Extensions, Normal and Separable Extensions, Solvable Groups, and Solvability of polynomial equations by radicals, Finite fields, and Cyclotomic fields

Pre-requisites: Some basic definitions and results from the courses:

MTH 203, MTH 301 and MTH 302.

Syllabus:

- Definition of a ring, field and basic examples
- Polynomial rings, Gauss lemma, Irreducibility criteria
- Field extensions, Algebraic extensions and algebraic closures
- Splitting fields, Separable and Inseparable extensions
- Cyclotomic polynomials, Galois extensions
- Fundamental theorem of Galois theory
- Composite and Simple extensions, Abelian extensions over $\mathbb{Q}$
- Galois groups of polynomials, Solvability of groups, Solvability of polynomials
- Computations of Galois groups over $\mathbb{Q}$.
- Classical Straight hedge and compass constructions (optional)

Link for the official course syllabus:

$http://acad.iiserb.ac.in/cc/mth401.php$

Textbook:

- D. S. Dummit, R. M. Foote, Abstract algebra, 3 Ed., Wiley, 2004. (For problem solving)

Reference books:

- Stephen C. Newman, A Classical Introduction to Galois Theory.
- Nathan Jacobson, Basic Algebra.
- Emil Artin, Algebra with Galois Theory, Curant Lecture Notes.
- David A. Cox, Galois Theory (Pure and Applied Mathematics: A Wiley Series of Texts, Monographs and Tracts).
- I. Stewart, Galois Theory (Third Edition). Chapman and Hall/CRC, 2004. (Main module text.)
- Emil Artin, Galois Theory. Dover Publications, 1998.
- P.J. Cameron, Introduction to Algebra (Second Edition). Oxford University Press, 2007.
- I.N. Herstein, Topics in Algebra (Second Edition). John Wiley and Sons, 1975.
- J.S. Milne, Fields and Galois Theory.
  Available at http://www.jmilne.org/math/CourseNotes/FT.pdf.
- D.J.H. Garling, A Course in Galois Theory. Cambridge University Press, 1986.
- Steven H. Weintraub, Galois Theory (Universitext) Spinger.
- Edgar Dehn, Algebraic Equations: Introduction to the Theories of Lagrange and Galois.
- Josheph Rotman, Galois Theory (Universitext).
- Juliusz Brzezinski,Galois Theory Through Exercises (Springer Undergraduate Mathematics Series.
- Ramji Lal, Algebra 2: Linear Algebra, Galois Theory, Representation theory, Group extensions and Schur Multiplier (Infosys Science Foundation Series).
- I.S. Luthar, I.B.S. Passi, Algebra Vol 4 Field Theory (Narosa Publishing House).
- John M. Howie, Fields and Galois Theory-(Springer Undergraduate Mathematics Series)(2006).
- P.B. Bhattacharya, S. K. Jain, S.R. Nagpaul, Basic Abstract algebra, Cambridge University Press.

Assignment. There will be 7 assignments in this course and will be posted on the course webpage.

You are encouraged to work together on assignment problems, but **everyone has to write up the solutions independently**. Please order the pages and staple the pages. Unreadable homework will not be corrected. No late homework will be accepted.

Home work and class exercise. In every class you will get some home work which you don't need to submit. You can discuss it in office hours.

Grading Policy:

- **Participation of students in the course:** 10%**.**
  Students will be awarded marks for participation based on submission of assignments, quizzes, watching/listening/reading lecture material, interaction with faculty members etc. A minimum of 60% participation in the above components of a particular course is required for the students to be eligible for securing marks in this particular component.
- **Continuous assessment:** 70%**.**
  Continuous assessment during the entire duration of a course will be evenly spread out and consist of **take home quizzes** and **presentation**.
  4 synchronous/asynchronous assessments must be conducted. Best 3 out of 4 should be considered for final grading.
- **Final Assessment:** 20%**.**
  It includes Quiz/Surprize quizzes/Oral Exam/Assignments/Attendance/Class presentation etc.

***.** In case of any further questions regarding the course, please send me an email.