FIELDS AND GALOIS THEORY (MTH 401)
ASSIGMENT-1

SUBMISSION DATE: 28/08/2020

Problem-A. Submit an answer of all problems.

(1) Let F be a finite field of order q and let $f(x)$ be a polynomial in $F[x]$ of degree $n \geq 1$. Prove that $F[x]/(f(x))$ has q^n elements.
(2) Prove that if $f(x)$ and $g(x)$ arc polynomials with rational coefficients whose product $f(x)g(x)$ has integer coefficients, then the product of any coefficient of $g(x)$ with any coefficient of $f(x)$ is an integer.
(3) Let p be a prime and consider the polynomial $\phi_p(x) = 1 + x + x^2 + \cdots + x^{p-2} + x^{p-1}$. Show that it is irreducible polynomial.
(4) Show that the polynomial $x^4 + 10x + 5$ is irreducible over $\mathbb{Z}$.
(5) Show that the polynomial $(x-1)(x-2)\dots(x-n) - 1$ is irreducible over $\mathbb{Z}$ for all $n \geq 1$.
(6) Prove that $\mathbb{R}[x]/(x^2+1)$ is a field which is isomorphic to the complex numbers.
(7) Show that if a rational number $\frac{p}{q}$, where p, q are relatively prime integers, is a solution of an equation $a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n x^n = 0$ with integer coefficients a_i, then p divides a_0 and q divides a_n.

Problem:B. You don't need submit it.

(1) Find all the monic irreducible polynomials of degree ≤ 3 in $(\mathbb{Z}/2\mathbb{Z})[x]$, and the same in $(\mathbb{Z}/2\mathbb{Z})[x]$.
(2) Show that if a reduction of a monic polynomial $f(x) \in \mathbb{Z}[x]$ modulo a prime number p is irreducible as a polynomial in $(\mathbb{Z}/2\mathbb{Z})[x]$, then $f(x)$ is irreducible in $\mathbb{Q}[X]$.
(3) Show that over every field there exists infinitely many irreducible polynomials.

Date: 21-08-2020.