COMPLEX VARIABLES (MTH 204)

DR. SANJAY KUMAR SINGH

Assignment-4

Submission and Discussion Date: 04-02-2023

Problem 1. Solve problem 2.b, 2.d,4.b, from Page-70-71, Section24, Chapter 2.

Problem 2. Solve problems 1.b, 1.d, 2.b, 7 from Page-76-77, Section-26, Chapter 2.

Problem 3. Solve problems 1, 4, 5 from Page-84-85, Section-29, Chapter 2.

Problem 4. Solve all problems.

(a) Suppose that v is a harmonic conjugate of u in a domain D and also that u is a harmonic conjugate of v in D. Show that both $u(x, y)$ and $v(x, y)$ must be constant throughout D.

(b) Show that v is a harmonic conjugate of u in a domain D if and only if $-u$ is a harmonic conjugate of v in D.

(c) Show that
   (a) $\exp(2 \pm 3\pi i) = -e^2$;
   (b) $\exp(z + \pi i) = -\exp z$.

(d) Find all values of z such that
   (a) $e^z = -2$;
   (b) $e^z = 1 + \sqrt{3}i$.

(e) (a) Show that if e^z is real, then Im $z = n\pi$ $(n = 0, \pm 1, \pm 2, \cdots)$.
   (b) If e^z is purely imaginary, what restriction is placed on z?

(f) Show that
   (a) Log $(-ei) = 1 - \frac{\pi}{2}i$;
   (b) Log $(1 - i) = \frac{1}{2}\ln 2 - \frac{\pi}{4}i$;
   (c) $\log e = 1 + 2n\pi i \quad (n = 0, \pm 1, \pm 2, \cdots)$;
   (d) $\log i = (2\pi + \frac{1}{2})\pi i \qquad (n = 0, \pm 1, \pm 2, \cdots)$.

Text Book: R. V. Churchill and J. W. Brown, Complex variables and applications, McGraw-Hill, 2003, **9th Indian Edition**.

Note: Assignment submission is not compulsory. If you submit the assignment, Tutor will check it and mark your mistakes. It will be very helpful in the examination.