QUIZ-2

Introduction to Groups and Symmetry (MTH 203)

30-10-2019

Total Marks: 10 **Time: 55 Minutes**

Solve all Problems.

Problem-A. For each of the following statements indicate whether it is **TRUE** or **FALSE**. You DO NOT need to provide justification for your answers in this problem. (5)

(1) Let H be a normal subgroup of a cyclic group G. Then G/H need not be cyclic.

(2) Let G be a finite group and $\phi : G \to G$ be a group isomorphism. If ϕ fixes more than half of the elements of a G, then it is the identity automorphism.

(3) Let G be the cyclic group of order 120. It has more then 2 subgroups of order 3.

(4) Let H be the subgroup of all isometries of $\mathbb{R}^2$ that fixes the origin. Then
$$H \cong O(2).$$

(5) An isometry of $\mathbb{R}^2$ is determined by its value on any three non-collinear points.

Note: A map $f : X \to X$ fixes an element $x \in X$ means $f(x) = x$.

Problem-B. Solve all questions.

(1) Let G be a group and $x, y \in G$. Let $z = xy \in Z(G)$, then show that x and y commute. (2)

(2) Consider the line
$$L = \{(x; y) \in \mathbb{R}^2 : 3x - y = 0\}.$$
Write down the reflection map $R_L : \mathbb{R}^2 \to \mathbb{R}^2$ corresponding to L. (3)