INTRODUCTION TO GROUPS AND SYMMETRY

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MTH 203

Mid Semester Practice Problem

Problem Set-2 (Permutation Group)

(1) Let σ and τ be two permutations in S_n. Show that $\sigma\tau\sigma^{-1}\tau^{-1}$ is even permutation.

(2) Let σ and τ be two permutations in S_n. Show that $\sigma\tau$ is an even permutation if and only if σ and τ are both even, or both odd.

(3) Express each of the following permutations and their inverse as a product of disjoint cycles. Also compute the order. Write them as product of transposition. Classify all even permutation.

- $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 4 & 6 & 3 & 7 & 2 & 9 & 5 & 8 & 10 \end{pmatrix}$.
- $(1,4,5)(5,3,2)(2,1,6) \in S_6$.
- $(1,2)(2,3)(4,3)(5,7)(2,4)(6,1) \in S_7$.

(4) Find the order of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 7 & 2 & 5 & 4 & 6 & 8 & 9 & 10 & 1 & 3 \end{pmatrix}$. For $\tau = (3,7,8)$, compute $\sigma\tau\sigma^{-1}, \tau\sigma\tau^{-1}$ and their order. Are they even permutation?

(5) What is the largest possible order of a permutation in S_{10}? Write down an explicit element of this order.

(6) Find number of all elements of order 4 in A_6.

(7) How many elements of order 5 are in S_7 ?

(8) Let $\sigma = (1,3,5,7,9) \in S_{10}$. Find the number of element in S_{10} which commutes with σ.

(9) Show that S_{11} contains no elements of order 16.

(10) Show that S_{10} has elements of order $10, 12$, and 14, but not 11 or 13.

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(11) Find the number of elements of order 16 in S_{16}.

(12) Find the number of elements of order 35 is S_{12}.

(13) Find the number of permutations in S_{10} commuting with a cycle of lenth 5.

(14) Find the number of even permutations in S_n which commute with a cycle of lenth r.

(15) Let H be a subgroup of S_n all of whose nonidentity permutations are odd. Show that $o(H) = 1$ or 2.

(16) Find the number of conjugacy classes of A_4 and A_5.

(17) Do A_n and S_n contain same number of conjugacy classes? Justify your answer.

(18) Show that $Z(S_n) = I, n \geq 3$.

(19) Show that $Z(A_n) = I, n \geq 4$.

(20) Show that S_n is generated by a transposition and a cycle of length n.

(21) In S_n prove that there are $\frac{1}{r}\frac{n!}{(n-r)!}$ distinct r cycles.

(22) Let $\sigma = (i_1, i_2 \ldots, i_k) \in S_n$ be a cycle of lenght k. Show that any permutation of the form $\sigma^j\tau$ where $\tau \in S_n$ fixes all $i_1, i_2, \ldots i_k$ and j is any interger commutes with σ.