INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203)
ASSIGNMENT-8

Submission Date: 20-11-2019.

Note: Solve all problems.

Problems:

(1) Let G be a group.
- If $g \in G$ is an element of infinite order in a group, then $g^k = g^l$ if and only if $k = l$.
- If g is an element of finite order n in a group, then $g^k = g^l$ if and only if n divides $k - l$ i.e. $k \equiv l \ mod(n)$.
- If G is cyclic then show that any subgroup of G is also cyclic.

(2) Let G be a finite group. Then the following conditions are equivalent:
- G is cyclic.
- For each positive integer d, the number of $g \in G$ such that $g^d = e$ is less than or equal to d.
- For each positive integer d, G has at most one subgroup of order d.
- For each positive integer d, G has at most $\Phi(d)$ elements of order d.

(3) Check weather the following statements are True or False. Justify your answer?
- G is a finite cyclic group of order n iff it has the unique subgroup order d for every divisor d of n.
- Let d be a divisor of n. The number of elements of order d in $\mathbb{Z}_n$ is $\Phi(d)$
- In a finite cyclic group, two elements generate the same subgroup if and only if the elements have the same order.
- Let G be a finite cyclic group. For subgroups H and K of G, $H \subset K$ if and only if $o(H)$ divides $o(K)$.
- Let G be a cyclic group of order n, generated by a. Then the order of a^k is $n/gcd(n, k)$. Note that any element is representted by a^i for

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some $i \leq n$.

- Number of all the elements of order 9 in $\mathbb{Z}_{9000}$ is 6.

  Note: $\Phi(d) =$ the number of elements which are coprime to d and less then d.