INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203)
ASSIGNMENT-6

Submission Date: 06-11-2019.

Note: Solve all problems.

Problems:A.

(1) Consider the lines

$$L_1 = \{(x; y) : x + y = 2\}; L_2 = \{f(x; y) : x - y = 2\}.$$

Write down the reflection map corresponding to L_1, L_2 and find the effect on the point $P = (1; 0)$ of the reflections R_{L_1} and R_{L_2}.

Compute the composition map $R_{L_2} \circ R_{L_1}$ and $R_{L_1} \circ R_{L_2}$ explicitly and compare them.

(2) Show that the only isometry of $\mathbb{R}^2$ fixing $(0, 1)$, $(1, 1)$ and $(0, 0)$ is the identity.

(3) Show that

$$O(2) = \{A \in GL_2(\mathbb{R}) : Av.Aw = v.w \text{ for all } v, w \in \mathbb{R}^2\}.$$

Note that in the class, we have defined

$$O(2)(\mathbb{R}) = \{A \in GL_2(\mathbb{R}) : AA^t = I\}.$$

(4) Let $h : \mathbb{R}^2 \to \mathbb{R}^2$ be an isometry which fixes the origin. Show that h preserves angles between two lines.

(5) Define $\phi : \mathbb{R}^2 \to \mathbb{R}^2$ by,

$$\begin{bmatrix} x \\ y \end{bmatrix} \mapsto \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = (xcos\theta - ysin\theta, xsin\theta + ycos\theta).$$

Show that ϕ is an isometry.

Problem-B:. No need to submit.

(1) Find nontrivial (non identity) examples of isometry of $\mathbb{R}^2$ which fixes the point $(0, 1)$.

(2) Find nontrivial (non identity) examples of isometry $h : \mathbb{R}^2 \to \mathbb{R}^2$ such that $h(S^1) = S^1$, where S^1 is the unit circle in $\mathbb{R}^2$.

$$S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}.$$