INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203)
ASSIGMENT-4

SUBMISSION DATE: 11-09-2019

GROUP-A

Problems for submission.

(1) Let G be a group. If H is a subgroup of G and N is a normal subgroup of G, show that $H \cap N$ is a normal subgroup of H. Also show that NH is a subgroup of G.

(2) Prove that the center of a group is always a normal subgroup.

(3) Prove that a group of order 9 is abelian.

(4) Let $f : G_1 \to G_2$ be a group homomorphism. Suppose G_1 is abelian (cyclic) show that $Im(f)$ is also abelian (cyclic).

(5) Let G be the group of order 9 generated by elements a, b where $a^3 = b^3 = e$. Find all automorphisms of G.

(6) Find all group homomorphisms from $\mathbb{Z}_{10} \to U_{15}$.

GROUP-B

You don't need to submit the following problems. You are encourage to discuss your solution with tutors.

(1) If p is a prime number, prove that any group G of order $2p$ must have a subgroup of order p, and that this subgroup is normal in G.

(2) Find all group homomorphisms from $\mathbb{Z}_{10} \to S_6$.

(3) Find all group homomorphisms from $S_3 \to U_{15}$.

(4) Find all group homomorphisms from $m\mathbb{Z} \to n\mathbb{Z}$.

(5) Find all group homomorphisms from $\mathbb{Z}_m \to \mathbb{Z}_n$.

(6) If G is an abelian group and n a positive integer, show that $\phi : G \to G$ defined by $g \mapsto g^n$ is a group homomorphism.

(7) Find all automorphisms of $U_5, U_{20}, S_3, \mathbb{Z}_{10}$.

(8) Let $H \subset G$ be a subgroup. Suppose that for any $a \in G$ there exists $b \in G$ such that $aH = Hb$. Show that H is normal in G.

(9) Let G_1, G_2 are two groups. Can you give a group structure on the set of all group homomorphism from G_1 to G_2. Try if G_1 or G_2 abelian group.