INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203) ASSIGMENT-3

SUBMISSION DATE: 04-09-2019

GROUP-A

Problems for submission.

(1) Let S_4 be the group of permutations on the set $\{1, 2, 3, 4\}$ and

$$H = \{(1, 2, 3, 4), (1, 3, 2, 4), (2, 1, 3, 4), (2, 3, 1, 4), (3, 1, 2, 4), (3, 2, 1, 4)\}$$

be a subset of G. Show that H is a subgroup of S_4. Compute the left and right cosets of H in G. Is H normal subgroup of S_4?

(2) Suppose that H is a subgroup of G such that whenever $Ha \neq Hb$ then $aH \neq bH$, where $a, b \in G$. Prove that $gHg^{-1} \subset H$ for all $g \in G$.

(3) How many generators does a cyclic group of order n have?

(4) Can you find a generator of the group U_{20}? Is U_{20} cyclic?

Note: If U_{20} have no single generator then find a minimal generating set (smallest subset of U_{20} which generates U_{20}). See class notes for a definition of U_{20}.

(5) Let G be a group and H, K are two subgroups of finite index in G, prove that $H \cap K$ is of finite index in G. Can you find find an upper bound for the index of $H \cap K$ in G?

(6) Let $GL_2(\mathbb{Z}_2)$ = The set of all 2×2 matrices with entries in $\mathbb{Z}_2$. Show that the order of $GL_2(\mathbb{Z}_2)$ is 6. Compute the order of each element. Show that it is non abelian group. Note that any element in $GL_2(\mathbb{Z}_2)$ can be written as,

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

where $ad - bc \neq \bar{0}$ and $a, b, c, d \in \mathbb{Z}_2 = \{\bar{0}, \bar{1}\}$. Try above problem over $\mathbb{Z}_3$.

GROUP-B

Problems you don't need to submit.You are incourage to discuss your solution with any tutors.

(1) Let G be any group and H be a subgroup of G. Show that number of left cosets of H is same as number of right cosets of H in G. Hint: See class notes.

(2) Let a, b are two non zero positive interger and d is the gcd of them. Show that there exist intergers q, r such that,

$$d = aq + br.$$

Is converse of the above statement true? Hint: See Lemma 3.1 in Herstein Chapter-1.

(3) Let a, b, c are non zero intergers and $(a, b) = 1$ prove that $(ab)|c$.

(4) Let G be a group and H be a subgroup of G. Let

$$N = \bigcap_{x \in G} xHx^{-1}.$$

Prove that N is a normal subgroup of G.

(5) Solve the following problems from Herstein (2nd edition) Book.
- Problem $24, 25, 29, 34, 36$ in Chapter-2, page 48, 49.
- Problem $8, 9, 14, 21$ in Chapter-2 section-6 on page 53, 54.

Note: Some of students know how to solve problems but not abel to write it mathematically. Tutors will solve few problems from assignment in the class. It will help you to get idea about writing solution properly.

Assignment grading will be very strict such that you can find your mathematical mistakes