INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203)
ASSIGMENT-2

SUBMISSION DATE: 21-08-2019

GROUP-A

Problem-1.

(1) List all the possible equivalence relations on the set $X = \{a, b\}$.

(2) Let X be a set and $\sim$ be an equivelence relation (i.e. a subset A of $X \times X$) on X. Show that if $a \sim b$ then $[a] = [b]$ and if c is not releted to d then $[c]$ and $[d]$ sre disjoint. where $a, b, c, d \in X$.

(3) Let n be an integer, define $A_n = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} : a - b = kn$ for some $k \in \mathbb{Z}\}$. Show that A gives an equivalence relation $\sim_n$ on $\mathbb{Z}$. Find the equivalence classes [2] and [3]. Also find the set of equivalence classes $\mathbb{Z}/\sim$ can you give a group structure on $\mathbb{Z}/\sim$ using + and . operation on $\mathbb{Z}$.

(4) Let G be a finite group. Show that there exist no non trivial group homomorphism from $G \to \mathbb{Z}$. Here $\mathbb{Z}$ is a group with respect to addition of integers.

(5) Let $X = \{1, 2, 3\}$ define a set S = Set of all bijective map from $X \to X$. Define a map

$$\mu : S \times S \to S, (f, g) \to g \circ f$$

where $g \circ f$ is composition map of f with g. Show that S is a non abelian group of order 6 with respect to the binary operation μ.

GROUP-B

Problem-1:

(1) Prove that any subgroup of cyclic group is cyclic.

(2) Let G be a finite cyclic group and a be a non identity element in G show that $o(a)$ divide $o(G)$. Note $o(a)$ denote order of a and $o(G)$ denotes number of elements in G. Is it true for any finite group G?

(3) If in the group G, $a^5 = e, aba^{-1} = b^2$ for some $a, b \in G$, find $o(b)$.

(4) Let G be any group, g a fixed element in G. Define $\psi : G \to G$ by $\psi(x) = gxg^{-1}$. Show that ψ is group homomorphism and bijective.

(5) Let G be an abelian group. Show that,
- Centre $Z(G) = G$,
- $C_G(A) = N_G(A) = G$ for any subset A of G. Note here $C_G(A)$ and $N_G(A)$ denotes centralizer and Normalizer of A in G.

(6) Let $G = S$ group of all bijective maps from $\{1, 2, 3\} \to \{1, 2, 3\}$ defined above. Let $A = \{f\}$ where $f \in S$ and $f(1) = 2, f(2) = 3, f(3) = 1$ find $Z(G), C_G(A)$ and $N_G(A)$. Take any subset A of G and compute $C_G(A)$ and $N_G(A)$.

Date: 14-08-2019.

Note: 1. Group B is optional. You don't need to submit it. It will be discussed in tutorial.
2. Most of the problems are very standard and easily found in any group theory book. I have taken most of the problems from Herstein's Book.