INTRODUCTION TO GROUPS AND SYMMETRY (MTH 203)
ASSIGMENT-1

SUBMISSION DATE: 14-08-2019

GROUP-A

Problem-1. Let G be a group.

(1) Show that the intersection of any two subgroups of G is also a subgroup of G. By an example show that the union of two subgroups of G need not be subgroup?
(2) If $(a.b)^2 = a^2.b^2$ for all $a, b \in G$, show that G must be abelian.
(3) Show that if every element of the group G is its own inverse, then G is abelian.
(4) If G has no nontrivial subgroups, show that G must be finite of prime order.
(5) If order of G is prime number then show that G is cyclic group.

Problem-2. Find all group homorphisms from $(\mathbb{Q}, +)$ to $(\mathbb{Q}, +)$.

GROUP-B

Problem-1:

(1) Let $M(n, \mathbb{R})$ denote the set of all $n \times n$ matrices whose entries are real number. For $A, B \in M(n, \mathbb{R})$ Define a binary operation $A \circ B = A + B$,where $A + B$ is the usual matrix addition of A and B. Show that with respect addition $M(n, \mathbb{R})$ is a group. What can you say about for the binary operation $A \circ B = A.B$ where $A.B$ is the usual matrix multiplication of A and B.
    Define $GL(n, \mathbb{R}) = \{A \in M(n, \mathbb{R}) : det(A) \neq 0\}$.
    Is $GL(n, \mathbb{R})$ a subgroup of $(M(n, \mathbb{R}), +)$?
(2) Show that $GL(n, \mathbb{R})$ is a group with respect usual multiplication of matrices. and $det : GL(n, \mathbb{R}) \to GL(1, \mathbb{R}) = \mathbb{R}^*$ is a group homomorphism.
(3) Let G be a finite abelian group. Prove that if order of G is odd, then every element has a unique square root; that is, for $x \in G$ there exists exactly one $g \in G$ with $g^2 = x$.
(4) Suppose a finite set G is closed under an associative product and that both cancellation (right and left cancellation)law holds in G. Prove that G must be a group.

Note: 1. Group B is optional. You don't need to submit it. It will be discussed in tutorial.
2. Most of the problems are very standard and easily found in any group theory book. I have taken most of the problem from Herstein Book.

Date: 07-08-2019.