MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS (MTH-201)

QUIZ-3 (11/11/2016)

Time: 60 minutes   Maximum Marks: 20

Attempt all questions. Use separate page for each answer.

Problem 1.

(1) Let S be an oriented surface bounded by a smooth curve C. Let $\mathbf{F}$ be a vector field on an open set containing the surface and its boundary, and assume that $\mathbf{F}$ is perpendicular to the boundary(i.e. at every point of the boundary, the value of the vector field is perpendicular to the tangent line of the curve). Show that

$$\iint_S (\nabla \times \mathbf{F}) \bullet \mathbf{n} d\sigma = 0$$

where $\mathbf{n}$ is the orientation (i.e. the normal vector at every point of the surface) of the surface.

(2) Verify the Divergence Theorem for the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ over the cube defined by $0 \leq x \leq 1, 0 \leq y \leq 1$ and $0 \leq z \leq 1$.   (4+8)

Problem 2. Solve the following ordinary differential equations.

(1) $\left(x + \frac{y}{2}\right) dy + y \, dx = 0$
(2) $e^x(x+1)dx + (ye^y - xe^x)dy = 0$.   (3+5)