Multivariable Calculus and Differential Equations (MTH-201)

Quiz 2 (25/10/2016)

Time: 60 minutes Maximum Marks: 16

Marks for each questions are given right side.

Problem 1. (1+3)

(A) Give the statement of Green's theorem in the plane. Clearly write all assumption needed in the theorem.

(B) Find the counterclockwise circulation and outward flux of the vector field $F = xy\,\hat{\mathbf{i}} + y^2\,\hat{\mathbf{j}}$ around and over the boundary of the region enclosed by the curves $y = x^2$ and $y = x$ in the first quadrant.

OR

Show that the value of

$$\oint_C xy^2 dx + (x^2 y + 2x) dy$$

around any square depends only on the area of the square and not on its location in the plane.

Problem 2. Let C be a smooth curve in xy plane satisfying the equation $x^2 + xy + y^2 = 1$ joining the point $(0,1,0)$ to $(1,0,0)$. Compute the line integral $\int_C \mathbf{F} \bullet d\mathbf{r}$, where $\mathbf{F} = e^{y+2z}(\hat{\mathbf{i}} + x\,\hat{\mathbf{j}} + 2x\,\hat{\mathbf{k}})$. (3)

Problem 3. Let f be defined on the rectangle $Q = [0,2] \times [0,2]$ as follows:

$$f(x,y) = \begin{cases} 0, & \text{if } x = y \\ 10, & \text{otherwise.} \end{cases}$$

Using a definition of double integrals show that $\iint_Q f$ exists. Find $\iint_Q f$?

Note: In class and assignment, I have given 3 equivalent definition of double integrals using partitions. You can use anyone. (4)

Problem 4. Let R be the region in the first quadrant of the xy-plane bounded by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$ and $y = x + 1$. Evaluate

$$\iint_R (2x^2 - xy - y^2) dA.$$

Hint: Use suitable transformation. (5)