MTH 201

MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS

QUIZ 1 (26/08/2016)

Time: 50 minutes   Maximum Marks: 7

Marks for each question is given right side.

Problem 1. Are the following statements true or false? **Do not** give any proof for each statement.

(A) If three vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ intersect at the origin are in a common plane, then $\mathbf{u}.((\mathbf{v} + \mathbf{u}) \times \mathbf{w}) = 0$.

(B) Let $\mathbf{r}(t)$ be a space curve such that speed 1 and curvature 1 everywhere, then $\mathbf{r}(2t)$ has speed 2 and curvature $1/2$ everywhere.
   Note: Speed $= |d\mathbf{r}/dt|$ (1/2+1/2)

Problem 2. Let $P = (1, 1, 1), Q = (0, 3, 1)$ and $R = (0, 1, 4)$ are point in $\mathbb{R}^3$.

(A) Find the plane through P, Q and R, expressed in the form $ax + by + cz = d$.

(B) Is the line through $(1, 2, 3)$ and $(2, 2, 0)$ parallel to the plane in part (A) ? Explain why or why not.   (2+1)

Problem 3. Solve any two.   (3/2+3/2)

(A) Give the definition of a smooth curve in 3 space. Is the curve $\mathbf{r} : [-1, 1] \to \mathbb{R}^3$,

$$t \mapsto (t^2 - 1, cos\ t, t^3 - 1).$$

   smooth? Give a reason.
      Note: We can also write above curve as, $\mathbf{r}(t) = (t^2 - 1)\mathbf{i} + cos\ t\ \mathbf{j} + (t^3 - 1)\mathbf{k}$, where $t \in [-1, 1]$.

(B) Find the torsion of the curve $y = x^4$, $z = 0$.

(C) Find the curvature for the helix,

$\mathbf{r}(t) = (a\ cos\ t)\mathbf{i} + (a\ sin\ t)\mathbf{j} + bt\mathbf{k}$, $a, b \in \mathbb{R},\ a, b \geq 0,\ a^2 + b^2 \neq 0$.