# MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS (MTH 201)

Mid Semester Examination (18/09/2016)

Time: 120 minutes Maximum Marks: 30

**Problem A.** Are the following statements true or false? **Do not** give any proof for each statement.

(a) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function and $\mathbf{u}$ be any vector in $\mathbb{R}^2$. If $\partial f/\partial x, \partial f/\partial y$ exists and continuous at each point then the directional derivative $f'(\mathbf{u}, \mathbf{a})$ and $f$ are also continuous.

(b) Any continuous map from a bounded subset of $\mathbb{R}^n$ to $\mathbb{R}$ is bounded.

(c) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function of two variable $x$ and $y$. The tangent plane at point of local maxima or minima is always parallel to $xy$ plane.

(d) Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Let $a, b \in \mathbb{R}$ such that $f(a) \neq f(b)$, then for every number $w$ in between $f(a)$ and $f(b)$, there exist $r \in \mathbb{R}$ such that $f(r) = w$. (1+1+1+1)

**Problem B.** (3+3+3+3)

(1) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defined as,

$$(x, y) \to \frac{e^{(sinx+cosy)}.\sqrt{(x^2+y^2)}}{log(x^2+y^2+2)}.$$

Show that $f$ is a continuous function.

(2) Let $A$ be a closed and open subset of $\mathbb{R}$. Define $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A. \end{cases}$$

Show that,

(1) $f$ is continuous,

(2) $A$ is either empty set or $\mathbb{R}$.

(3) Find a constant $c$ such that at any point of intersection of the two spheres

$$(x-c)^2 + y^2 + z^2 = 3 \text{ and } x^2 + (y-1)^2 + z^2 = 1$$

the corresponding tangent planes will be perpendicular to each other.

(4) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function. If $f(0,0) = 0$ and $f(tx, ty)) = tf(x, y)$ for all $t \in \mathbb{R}$ and $(x, y) \in \mathbb{R}^2$, prove that $f(x, y) = \bigtriangledown f(0, 0) \bullet (x, y)$ for all $(x, y)$. In particular $f$ is linear. Note $\bullet$ is dot product.



*P.T.O.*

**Problem C.** (5+5+5)

(5) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a differentiable function and $\partial f/\partial y = 0$. Show that $f$ is independent of the second variable. If $\bigtriangledown f = 0$, show that $f$ is constant.

(6) Show that the maximum value of the function $f(x, y, z) = x^2y^2z^2$ on the sphere $S =: x^2 + y^2 + z^2 = 1$ is $(1/3)^3$. Also show that for any positive number $a_1, a_2, a_3$,

$$(a_1a_2a_3)^{1/3} \leq (a_1 + a_2 + a_3)/3.$$

(7) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defines as,

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Show that $f$ is continuous.

(b) Show that partial derivatives $\partial f/\partial x, \partial f/\partial y$ at $(0, 0)$ exist and equal to zero 0.

(c) Show that directional derivatives exist at $(0, 0)$.

(d) Show that $f$ is not differentiable at $(0, 0)$.