MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS (MTH-201)

END SEMESTER EXAMINATION (04/12/2016)

Time: 180 minutes Maximum Marks: 100

Marks for each question are given right side. Explain each step clearly.

(1) Show that the function
$$f(x,y) = \frac{xy^2}{x^2+y^4}$$
has no limit as (x,y) approaches $(0,0)$. (6)

(2) Find the directional derivative of a function $f(x,y,z) = e^x y + e^z y^2$, in the direction of vector $\mathbf{u} = \frac{1}{3}(2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ at the point $(1,0,2)$. (6)

(3) Let $X = \{(x,0) : 0 \leq x \leq 1\}$ be a subset of $\mathbb{R}^2$. Find the set of interior, exterior and boundary points of X. Is X closed? (6)

(4) Find the equation of the tangent plane and normal line to the surface
$$x^2 + y^2 - 2xy - x + 3y - z = -4$$
at the point $(2,-3,18)$. (6)

(5) Solve the following differential equation, (8)
$$(e^x - siny)dx + cosy\ dy = 0.$$

(6) State the Fubini's theorem. Calculate (3+5)
$$\int_0^{\pi/2} \int_x^{\pi/2} \frac{sin\ y}{y} dydx.$$

(7) Using Lagrange Multipliers method find the maximum and minimum values of
$$f(x,y,z) = x - 2y + 5z$$
on the sphere $x^2+y^2+z^2 = 30$. (10)

(8) Evaluate
$$\int_0^1 \int_0^{1-x} (y-2x)^2 \sqrt{(x+y)}\ dydx.$$
Also sketch the region of integration.
Hint: Use transformation $u = x + y$ and $v = y - 2x$. (15)

(9) Apply the ϵ-δ definition of limit to find
$$lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2+y^2}$$
if it exists. (15)

(10) Verify the Gauss divergence theorem for the vector field $\mathbf{F} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ over the sphere of radius 3 centered at the origin. (20)

• Do not use any formula for area and volume in this question. If you needed then compute it.