MTH 201

MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS

Assignment-6

Problem-1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function defines as,

$$f(x,y) = \begin{cases} \frac{x^3 y}{x4+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Show that f is continous. Hint: use arithmatic mean - geometric mean inequality and Sandwich theorem.
(b) Show that partial derivatives at $(0,0)$ exist and equal to zero 0.
(c) Show that directional derivatives at $(0,0)$ is written as linear combination of partial derivatives.
(d) Show that f is not differential at $(0,0)$.
(e) Can you jusify why f is not differential at $(0,0)$? Here each directional derivatives at $(0,0)$ are written as linear combination partial derivatives at $(0,0)$. Justify yourself?

Problem-2. Solve 33, 34, 38, 40, 41, 42, 43 from Section 14.8

Problem-3. Read the proof of second derivative test from section 14.10.

Note: If you have any doubt in your solution then you can discuss it in tutorials.

Text Book: Thomas' Calculus 11th edition (Maurice D. Weir, Joel Hass, Frank R. Gioedano).