MTH 201

MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS

Assignment-5, Solution discussion date 16/09/2016

Problem-1. Solve 5, 8 from Section 14.6

Problem-2. Solve 31, 70 from Section 14.7

Problem-3. Solve 10, 28, 31, 38, 45, 50 from Exercises 14.4

Problem-4. Solve 27, 31, 32, 36 from Excercise 14.5

Problem 5. If $f : [a, b] \times [c, d] \to \mathbb{R}$ is continous and $\partial f/\partial y$ is continous, define

$$F(x, y) = \int_a^x f(t, y)dt.$$

- Find $D_x F$ and $D_y F$.
- If $G(x) = \int_a^{g(x)} f(t, x)dt$, find $G'(x)$.
  Hint: Use fundamental theorem of calculus and Chain rule.
  If $F(x) = \int_a^x f(t)dt$ then $dF/dx = f(x)$.

Problem 6. If $f : \mathbb{R}^2 \to \mathbb{R}$ and $D_2 f = 0$, show that f is independent of the second variable. If $D_1 f = D_2 f = 0$, show that f is constant.

Problem 7. Let $f : R^3 \to R$ and $g : R^2 \to R$ be differentiable. Let $F : R^2 \to R$ be defined by the equation

$$F(x, y) = f(x, y, g(x, y)).$$

(a) Find DF in terms of the partials of f and g.
(b) If $F(x, y) = 0$ for all (x, y), find $\partial g/\partial x$ and $\partial g/\partial y$ in terms of the partials of f.

Problem 8. If $f : \mathbb{R}^3 \to \mathbb{R}$ is differentiable and $f((0, 0, 0)) = 0$, prove that there exist $g_i : \mathbb{R}^3 \to \mathbb{R}$ for $i = 1, 2, 3$ such that

$$f(x) = xg_1(x, y, z) + yg_2(x, y, z) + zg_3(x, y, z)$$

Problem 9. Show that if $f : \mathbb{R}^3 \to \mathbb{R}$ is differentiable at $a \in \mathbb{R}^3$ there is a **unique** linear map $T_a : \mathbb{R}^3 \to \mathbb{R}$ such that

$$lim_{h\to 0} \frac{|f(a+h) - f(a) - T_a(h)|}{|h|} = 0$$

where $h = (h_1, h_2, h_3)$ and $|h| = (h_1^2 + h_2^2 + h_3^2)^{1/2}$

Problem 10. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function such that $|f(x)| \leq |x|^2$. Show that f is differentaible at 0. Remark: $x = (x_1, x_2)$ and $|x| = (x_1^2 + x_2^2)^{1/2}$

Note: If you have any doubt in your solution then you can discuss it in tutorials.

Text Book: Thomas' Calculus 11th edition (Maurice D. Weir, Joel Hass, Frank R. Gioedano).