MTH 201

MULTIVARIABLE CALCULUS AND DIFFERENTIAL EQUATIONS

ASSIGNMENT-11 (11/11/2016)

DISCUSSION DATE: (18/11/2016)

Problem:A.

(1) Prove or disprove that if $\nabla \bullet \mathbf{F} = 0$ and $\nabla \times \mathbf{F} = 0$, then $\mathbf{F} = 0$.
(2) Show that there exist no smooth vector field $\mathbf{F}$ such that $\nabla \times \mathbf{F} = z\ \mathbf{k}$.
(3) Verify the divergence theorem for the vector field $\mathbf{F} = 7x\mathbf{i} - z\mathbf{k}$ over the region in $\mathbb{R}^3$ bounded by the sphere $x^2 + y^2 + z^2 = 4$.
(4) Let $F = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$. Prove that
$$\frac{1}{2}\nabla(F.F) = F \times (\nabla \times F) + (F.\nabla)F,$$
where $F.\nabla = M\frac{\partial}{\partial\ x} + N\frac{\partial}{\partial\ y} + P\frac{\partial}{\partial\ z}$.
(5) Let f be a scalar function on an open set of $\mathbb{R}^3$. Find the condition on f such that
$$curl\ grad\ f = 0.$$

Problem:B. Solve the following differential equation.

(1) $x^3\ dy + (3yx^2 - sin\ x)\ dy = 0,\ x \neq 0.$
(2) $dy + (y - xy^3)\ dx = 0.$
(3) $tan\ \theta\ dr - (r + tan^2\theta)d\theta = 0.$
(4) $x^2(x-1)\ dy - (y^2 + x(x-2)y)\ dx = 0.$

Note: You can discuss your solutions in tutorials.