Statistical Physics and Solid State Physics (PHY 201)
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Assignment 7 (9-11-2010)

1. Consider a simple harmonic oscillator in one dimension. If the oscillator is allowed to have energy between E and $E+\Delta E$ draw in a phase space diagram allowed phase space *volume* of the oscillator.

2. Consider a simple pendulum. The bob of the pendulum starts from the end point $x = a$ and due to air drag (and friction at the point of suspension of the wire) the bob comes to rest at $x = 0$ after sometime. Draw the phase space diagram for the bob of the pendulum.

3. Show that the average kinetic energy per particle in a system of particles obeying Maxwell-Boltzmann distribution is $\frac{3}{2}kT$.

4. Consider a system of particles that follow Maxwell-Boltzmann distribution. Assume that particles with velocities in the range v to $v + dv$ have energies in the range ϵ to $\epsilon + d\epsilon$.
   a) Find the number of particles $n(v)dv$ in the velocity range v to $v + dv$.
   b) Define the *most probable* velocity as the velocity of the maximum number of particles. Can you find the *most probable* velocity of the particles.
   c) Find the *average velocity* of the particles.

5. Assume that air molecules in a room at 27° Celcius follow Maxwell Boltzmann distribution. Find average kinetic energy of one air molecule. Convert it in the unit of electron volt. How does this result compare with the ionization energy of the hydrogen atom? Calculate the *most probable* energy.

6. Calculate the number of (micro) states for a single particle whose energy lies between ϵ and $\epsilon + \Delta\epsilon$, where $\Delta\epsilon > 0$. Assume that the energy, ϵ, is related to momentum, p, following, $\epsilon^2 = m_0^2c^4 + p^2c^2$. Assume that the particles do not have spin. How does the result changes if the particles have spin s.

7. The radiation from so-called big-bang, Cosmic Microwave Background (nicknamed CMB) form an almost perfect blackbody distribution. The maximum of the energy density, $u(\lambda)$, of the radiation occurs at a wavelength of 1.1 mm. Find the temperature of the CMB. Compare your result with the experimentally observed result $2.725K \pm 0.001K$ by J. C. Mather.
   (**Note**: For discovery of blackbody nature of CMB J.C. Mather was awarded Nobel Prize in 2006 along with his colleague George Smoot.)

8. Show that, for a system of fermions the average energy per particle at $T = 0K$ is given by $\bar{\epsilon} = \frac{3}{5}\epsilon_F$, where ϵ_F = Fermi energy.

9. Justify the distribution followed by following systems –
   a) 10^{21} molecules of hydrogen atoms occupying a volume of 1 liter at T = 300 K.
   b) Electrons in a metal at T = 300 K. (Assume that electron density in a metal $\sim$

$10^{29} m^{-3}$.
c) Neutrons in a neutron star.
(**Note: -** A neutron star is a result of gravitational collapse of a *massive* star which has consumed all its nuclear fuel which otherwise prevents a gravitational collapse. These stars are primarily made of neutrons and has a density $\sim 5 \times 10^{17} kgm^{-3}$. Assume that temperature inside neutron star is 10^6 K. Neutrons are spin half particles.)

10. Consider a system of two particles. If the total number of *single-particle* states available is g, find the entropy of the system if the particles are a) bosons, b) fermions, c) classical.

11. Calculate ratio of electronic specific heat and ionic specific heat for a solid at T = 10 K and T = 300 K. Assume that, Debye temperature for the solid is $\theta_D = 426K$.

12. Show that in a one dimensional solid with a periodic lattice structure the wave function, $\psi(x)$, for electrons can be written as, $\psi(x) = e^{ikx} u_k(x)$, where $u_k(x)$ is a periodic function of period a and $k = \frac{2m\pi}{Na}$, $a =$ lattice parameter, $N =$ total # of lattice points, $m \in \{0, 1, 2, ..., N-1\}$.