Quantum Physics (PHY 201)
IISER Bhopal
Assignment 4 (03-09-2010)

1. Consider the time dependent Schrodinger equation for a free particle, $i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}$. Using this equation and the definition $\langle p_x\rangle = m\frac{d\langle x\rangle}{dt}$ show that $\langle p_x\rangle = -i\hbar\int_{-\infty}^{\infty}\Psi^*(x,t)\frac{\partial\Psi(x,t)}{\partial x}dx$.

2. Consider a one dimensional potential $V(x) = V(-x)$. Show that, if $u(x)$ is an eigen function of the time independent Schrodinger equation with energy eigen value E then $u(-x)$ is also an eigen function with the same energy eigen value E.

3. Consider particle in a box whose sides are at $x = \pm a$. Show that the particle cannot have negative energy states. Find the energy eigen values and normalized eigen functions.

4. Consider a free particle inside a one dimensional box of length L with $0 \leq x \leq L$. Show that, for any Δx satisfying $0 < \Delta x < L/2$, the normalized eigen function, $u_n(x)$, satisfies
   a) $u_n(L/2 + \Delta x) = u_n(L/2 - \Delta x)$, if n is odd,
   b) $u_n(L/2 + \Delta x) = -u_n(L/2 - \Delta x)$, if n is even.

   (**Note:** In case a) the eigen function remains invariant under a reflection around $x = L/2$, whereas in case b) the eigen function changes sign. The first type of eigen functions are known to be *parity even* and the second type is known to be *parity odd).*

5. Show that, the probability flux, $J(x)$, in one dimension satisfies, $\int_{-\infty}^{\infty}J(x)dx = \langle p_x\rangle/m$.

6. Consider a wave function, $\Psi(x,t)$, whose time-independent part is purely real. Assuming $\Psi(x,t) = \psi(x)e^{iEt/\hbar}$, where $\psi(x)$ is real, show that
   a) the probability flux for such wave function vanishes,
   b) the momentum expectation value, $\langle p_x\rangle = 0$.

7. Find the probability flux for the wave function, $\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty}A(p)e^{i(px-Et)/\hbar}dp$.