Quantum Physics (PHY 201)
Assignment 2 (13-08-2010)

1. A 100 MeV photon collides with a proton at rest. What is the maximum possible energy transfer to the proton?

2. A photon of frequency ν traveling along the +z direction collides with an electron of momentum p along the -z direction. The value of p happens to be $h\nu/c$. The photon after the collision emerges at an angle of 90° with respect to the z axis. What is the frequency of the emerging photon?

3. The diffraction pattern by a single slit follows $I(\theta) = I_0 \sin^2(\pi x)/(\pi x)^2$, where $x = a\sin(\theta)/\lambda$. (See the figure below for the experimental arrangement). The first minimum of the diffraction pattern occurs at $\theta_{min} = \lambda/a$, where θ_{min} is small. Assume that all electrons diffract within the range $+\theta_{min}$ to $-\theta_{min}$ and that each electron has a momentum p_0 along the path at the position of slit.

   a) Show that the spread in the y component of momentum is $\approx p_0\theta_{min} = p_0\lambda/a$. We can think of this spread as the uncertainty, Δp_y, in the y component of the electron momentum.

   b) Surely the uncertainty in the y component of the position of the electron is $\Delta y \approx a$. Using this information and the result in a) show that $\Delta y \Delta p_y \approx h$.

4. Consider an electron inside a hydrogen atom which has a radius $r = 5.1 \times 10^{-11} m$.

   a) Assuming the uncertainty in the location of the electron $\Delta x = 2r$ find the minimum uncertainty, Δp_x, in its momentum.

   b) The actual electron momentum inside the hydrogen atom, p_x must be at least Δp_x. However, assuming $p_x \approx \Delta p_x$, find the De Broglie wavelength and kinetic energy of the electron inside the hydrogen atom. How does the kinetic energy compare with the binding energy of an electron inside the hydrogen atom?

5. A wave function in one dimension is given by, $\Psi(x) = A(\sin(\pi x/L) + \sin(2\pi x/L))$ for $0 \leq x \leq L$ and $\Psi(x) = 0$ for all other values of x. Is $\Psi(x)$ a normalized wave function? If not, find the normalized wave function. Also find the probability of finding the particle in $0 \leq x \leq L/2$.