Statistical Physics (PHY 201)
IISER Bhopal
Assignment 6 (14-10-2010)

1. Consider a function of two variables, x and y, namely $f(x, y)$. Suppose we are interested in maximizing $f(x, y)$ subject to the constraint that another function, $\phi(x, y)$ must vanish at the point of maximum of $f(x, y)$. Show that, the maximum of $f(x, y)$ is given by the condition, $\nabla f = \lambda \nabla \phi$, where λ is a constant (called Lagrange's multiplier) and ∇ denotes the gradient operator in 2-dimension.

2. Consider a system of N *indistinguishable* particles. Assume that, these N particles are distributed over k energy levels. The i^{th} energy level contains g_i *single-particle states*. Each of the n_i particles in the i^{th} energy level has an energy ϵ_i. Assume that the total number of particles, N, and total energy E of the system are two constants. Also, assume that, each single-particle state can contain arbitrary number of particles. Show that, the most likely distribution of the particles amongst the k energy levels is given by,
$$n_i = \frac{g_i}{e^{\frac{\epsilon_i - \mu}{kT}} - 1} ,$$
where $i = (1, 2, 3, ..., k)$.
(Note: This distribution is called Bose-Einstein distribution.)

3. Repeat the above exercise assuming each single-particle state can contain at the most one particle and keeping all other conditions unchanged. Show that, the most likely distribution of the particles amongst the k energy levels is given by,
$$n_i = \frac{g_i}{e^{\frac{\epsilon_i - \mu}{kT}} + 1} ,$$
where $i = (1, 2, 3, ..., k)$.
(Note: This distribution is called Fermi-Dirac distribution.)