FRIENDLY ADVICE FOR NAVIGATING A MATHS PH.D.

Nothing that is written here is "correct" in any absolute sense. Nor is it complete.

1. The First Year

(1) Study as you would have in your M.Sc., but with more discipline. This is now your *job*.
(2) Understand what parts of Maths you like/dislike.
(3) Do some in-depth reading on subjects that you like that are not a part of your course. Ask the instructor if you don't know where to begin.
(4) Meet as many faculty as you can and talk to them about their work.
(5) Do at least one (if not two) reading courses in the area(s) of your interest with a potential advisor.

1.1. **Choose an Advisor.** Some factors to keep in mind while approaching a potential advisor are:

(1) You should be interested in their subject area.
(2) You should like/get along with them (professionally speaking).
(3) They should be open to taking students.
(4) They should be actively doing research and publishing.
(5) They should have had some experience mentoring BSMS/PhD students (This is optional). If they have such experience, meet their former students and ask about their experience.

2. Second Year: Starting Research

Assumption: You have chosen an advisor and have cleared the written comprehensive exam.

This is the first year of your life that you will not be taking classroom courses. You need to be mentally prepared for this jump.

2.1. **Being a Professional.**

(1) Establish a routine:
    - Come in at the same time everyday (preferably before lunch)
    - Work 8-10 hours (of which only 2-3 hours will feel productive).
    - Do some exercise and something social every day.
(2) Respond to emails in a timely and professional manner.

2.2. **Relationship with your advisor.** This is the first one-on-one professional relationship in your life. Your advisor will have a huge effect on your career for many years to come, so build the relationship with friendship, mutual respect, and a professional distance.

(1) Meet your advisor once or twice a week.
(2) Be punctual, courteous, and discuss your (mathematical) problems openly.
(3) Work hard, but if you feel that he/she is pushing you too hard, be open and speak your mind.

2.3. **Identify a Research Problem.** Your advisor will help you here, but you need to be actively involved.

(1) Read everything your advisor gives you.
(2) Collect many allied problems surrounding your reading. Do not just focus on one problem.
(3) Collect and play with examples (at least one for each definition you come across).
(4) Have a concrete question to answer. Don't just read theory for the sake of it.

This prepares you for the Oral Comprehensive Exam (do it before March of 2nd year).

2.4. **Start independent learning.**

(1) Learn something new everyday (even if it is just a small lemma/proof technique).
(2) Organize your learning: Keep notes of *everything*, and revisit them frequently.
(3) Maintain a calendar and use it as a "To-do" list of things you want to learn.
(4) Write down interesting ideas/proof techniques.
(5) Be precise in your learning - you can't learn everything, so focus on what you need.

2.5. **Solving a Research Problem.**

(1) Prove a simplified version of the problem with many unnecessary hypotheses. Then look at each hypothesis and see how it was used.
(2) At each stage, look for examples and counterexamples (this is hard).
(3) At each stage, isolate lemmas or partial results that may be independently useful.
(4) Investigate these partial results/lemmas from different points of view. Find multiple proofs (if possible).
(5) Compare these partial results to existing ones to guide your intuition.
(6) Explain what you have done to your advisor.

2.6. **Reading a Paper.**

(1) Start by scanning the paper to understand the major results (statement only).
(2) Understand the basic definitions and what they are trying to achieve. Construct examples (and counter examples) to each.
(3) Read the applications/examples to understand what the authors are trying to do.
(4) Read the main theorems and try to understand how the proofs work. You will need to use many lemmas (and theorems from other papers) as "black boxes".
(5) Slowly resolve each "black box". Some may not get resolved. This is OK.

Build an understanding of how the authors attacked the problem. This will help you with your thesis problem.

3. Third/Fourth Year: Your Thesis Problem(s)

In broad terms, continue to do everything that you did in the second year (especially sticking to your routine), but refine your methods as needed.

3.1. **Focused Problem-Solving.**

(1) Specialize your learning to focus on 'your problem'. Cut out all other distractions.
(2) Continue Literature survey: You should know *everything* that has been done to solve your problem.
(3) Subscribe to the arXiv mailing list. Skim through one/two new papers every week.
(4) Read reviews on MathSciNet. Learn the names of people in your area doing important work.
(5) Prove one new lemma (or theorem!) every day.
(6) Start attending conferences/workshops whenever they are available.

3.2. **Broaden your horizons.**

(1) Seek out and learn things outside of your specific research area.
(2) Attend seminars on any topic of interest (not just in your research area).
(3) Start giving talks to anyone who is willing to listen. Try to organize a group of like-minded people.

3.3. **Write a paper.** At each point in the process, discuss it with your advisor (and other co-authors, if any).

(1) Understand the content of the paper (in broad strokes) and make a template.
(2) Write down all the main theorems, then the lemmas.
(3) Check all the mathematics for accuracy.
(4) Only after doing all this must you write the introduction and acknowledgements. The introduction should contain:
    - A broad overview of the question you are addressing.
    - An overview of existing results (with citations).
    - A description of your results (you may mention the main theorems).
    - A description of the applications (connect it back to existing results).
(5) After writing the paper, list down journals you want to send them to. Aim a little higher than you think you should (your assessment of your own work is not accurate right after you have finished).
(6) While submitting, post it on the arXiv.
(7) After submission, ignore it for 3-5 months. Work on the next project.
(8) If a referee rejects the paper, improve on it as best you can and send it to another journal. Repeat until it finds a home.

3.4. **Dealing With Problems.**

(1) Burn-Out and Mental Fatigue:
    - Don't work 12-14 hours everyday. You will get bored and exhausted.
    - Learn to "blow off steam" (go to the gym, go running, or just play loud music). Do this frequently.
(2) Getting stuck on a problem for weeks/months:
    - Don't worry. This is totally normal. You have to work on it though.
    - Revisit the points in section 2.5.
    - Find other problems/interesting ideas to work on.
    - Speak to your advisor.
(3) Depression:
    - Seek help of a qualified professional.
    - Lean on your friends and try to build deeper relationships with them.

3.5. **Identify Your Future Goals.** Do you want to continue doing research? Do you want to teach? Do you want to move into 'industry'?

(1) To continue in research:
    (a) Find problems to work on in the future.
    (b) Broaden your horizons (see above).
    (c) Develop a "taste" for important problems that appeal to you.
(2) If you want to teach:
    (a) TA for as many courses as you can. Understand your strengths and weaknesses.
    (b) Find out places you can apply to (government colleges, private universites, etc.)
    (c) Start visiting these places and talking to the teachers there.
(3) If you want to move into industry:

(a) Identify what you want to do. Talk to someone who is already doing that.
(b) Understand what your core skills are and improve on those if need be.
(c) Learn to program and use computers well.

For more information, see https://marktomforde.com/academic/gradstudents/gradstudents.html

4. Fifth Year: Finishing Your Ph.D.

4.1. Give talks at Conferences/Workshops.

(1) Engage in social activity outside the talks.
(2) Understand the job market, your competition, and potential employers.
(3) Identify potential collaborators and get them excited about your work (and vice-versa).
(4) Don't underplay your achievements/work. Don't brag, but don't put yourself down either.

4.2. Apply for Academic Jobs.

(1) Ensure that all documents you send out are double-checked for typos.
(2) Each department has specific needs (research, teaching, administration, engagement with students, outreach, etc). *Every* document given below must be written afresh for departments of different types.
(3) You need to write the following documents. If possible, read other people's documents to understand what you need to do.
    (a) Curriculum Vitae (CV).
    (b) Cover letter: Make it specific for that job and don't be boring.
    (c) Teaching Statement: Answer the following questions:
        - Why do you want to teach?
        - When you go to a classroom, what is your goal?
        - How do you go about trying to achieve that goal? Give examples.
        - What are your future teaching goals?
    (d) Research Statement: Explain the following:
        - A brief overview of your research area.
        - Your contribution; one for each publication if you have multiples.
        - Future goals both with this problem(s) and other problems you have not yet tried.
    (e) Sometimes, you may need to write a Resumé - a short (one page maximum) write-up about why you are suitable for the job.
(4) Create a professional webpage and keep it updated.
(5) If you are putting references down on your resume, speak to the person and get their consent. Include one person from outside the department if you can, and one person who can attest to your teaching abilities.
(6) When you go for a job talk, give a talk tailored to that department. Also, ask beforehand who the audience in the talk will be (for instance, will students be there?)

5. Post-Doctoral Work

(1) Continue your thesis work and aim for one (more) publication from it.
(2) Start at least one new problem. Ideally, this is something you have already thought about during your PhD.
(3) Apply for a research grant.
(4) Teach a course, and showcase/improve your teaching skills.
(5) Help organize a conference if you can.

For more of such advice, have a look at Terence Tao's "Career Advice" https://terrytao.wordpress.com/career-advice/