# SPECTRAL THEOREM FOR NORMAL OPERATORS: PROJECT DESCRIPTION

## 1. Overview

In an first course in Linear Algebra, one encounters the *spectral theorem*, which states that every real symmetric matrix is diagonalizable. The goal of this project is to understand and prove a far-reaching generalization of this to operators on a Hilbert space. Specifically, we will prove that every normal operator on a separable Hilbert space is *diagonalizable*, in the sense that it is unitarily equivalent to a multiplication operator.

## 2. Tentative Schedule

At each stage of the project, you will need to be proficient with topics covered in MTH503 (Functional Analysis) up until that point.

(1) Banach algebras and Spectral Theory (from [Arveson] and [Notes])
  (a) Invertible elements, the spectrum of an element.
  (b) Commutative Banach algebras
  (c) Spectral permanence theorem

`[by October]`

(2) C*-algebras (from [Murphy], [Arveson] and [Notes])
  (a) Commutative C*-algebras
  (b) Continuous Functional Calculus

`[by December]`

(3) The Spectral Theorem (from [Notes])
  (a) Multiplication Operators and the first version of the theorem
  (b) Borel Functional Calculus
  (c) Spectral Measures and the second version of the theorem
  (d) Applications

`[by March]`

## References


[Arveson] W. Arveson, *A Short Course on Spectral Theory*
[Murphy] G. Murphy, *C*-algebras and Operator Theory*
[Davidson] K. Davidson, *C* Algebras by Example*
[Notes] Notes on Operator Theory and Operator Algebras, `http://home.iiserb.ac.in/~prahlad/documents/teaching/mth510_notes_2019.pdf`