TOPOLOGICAL DEGREE AND FIXED POINT THEOREMS : PROJECT DESCRIPTION

Topological Degree theory is concerned with the following problems: Let $\Omega \subset \mathbb{R}^n$ be an open set, and $f : \Omega \to \mathbb{R}^n$ be a continuous function. Given a point $y \in \mathbb{R}^n$, we are interested in solutions of the equation $f(x) = y$. Specifically, we would like to know if such solutions exist, and if so, we would like to understand their distribution in Ω. Furthermore, we would want to understand how these answers change if we perturb the function f.

Taking inspiration from the winding number of an analytic function, we begin by understanding the Brouwer mapping degree [Deimling], which associates a number to the above situation, denoted by $d(f, \Omega, y)$. We will study how this is defined, and show that, under certain conditions, it is unique. We then use it to prove some important theorems in topology, including the Brouwer Fixed point theorem, the Borsuk-Ulam theorem, and Jordan's separation theorem.

We then turn to the infinite dimensional version of the above question, where $\mathbb{R}^n$ is replaced by an infinite dimensional Banach space. The analogous notion of degree there is called the Leray-Schauder degree. We study this object, and its properties, and use it to prove the Schauder fixed point theorem, and to prove existence and uniqueness theorems for certain ordinary and partial differential equations ([Teschl] or [Fonseca,Gangbo]).

Time permitting, we may also discuss the Brower-Kronecker degree for maps between manifolds and the Hopf theorems [Outerlo, Ruiz].

References

[Deimling] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag (1985)
[Fonseca,Gangbo] I. Fonseca, W. Gangbo, *Degree Theory in Analysis and Applications*, OUP (1995)
[Teschl] G. Teschl, http://www.dim.uchile.cl/~chermosilla/NonlinearFunctionalAnalysis.pdf
[Outerlo, Ruiz] E. Outerlo, J.M. Ruiz, *Mapping Degree Theory*, AMS (2009)

Important Comments

(1) We will have weekly meetings in which you are expected to give a progress report and discuss questions. You must treat these meetings with utmost seriousness, and must cancel them only in case of unavoidable emergencies. (Taking the GATE/GRE do not count as emergencies)
(2) Every other week or so, you will be expected to give a short board presentation on what you have learnt.
(3) You should start writing your project report immediately after completing the first topic. I expect to see the first draft of Chapter I by the end of Semester IX.
(4) Read the DUGC guidelines for the project, and pay attention to the deadlines given therein. If you have any questions, make sure that you ask them *now*.
(5) Please make sure that you complete all the paperwork on time, and ensure that you do not rush your PEC members by doing things last-minute.