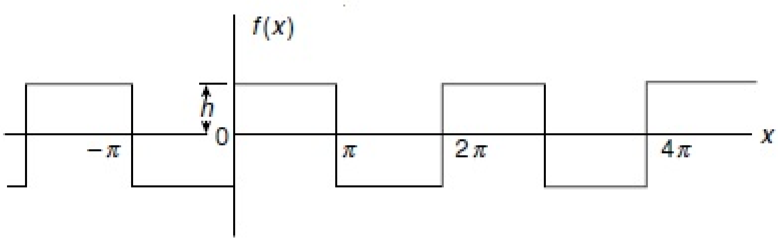
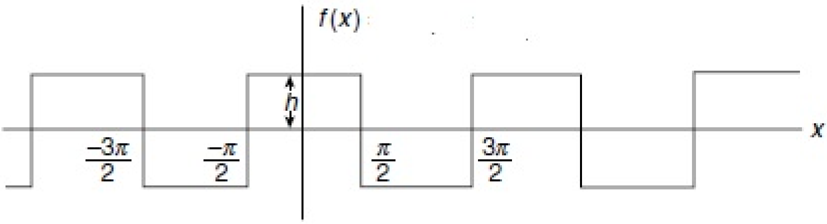
PHY201 Waves and Optics
Assignment 5

Handed out: September 09, 2018 Discussion: September 13, 2018

Problem 1





For the square waves shown in the figures above, calculate the coefficients of the Fourier series representation.

Problem 2

When you clap your hands periodically, the resulting air pressure at your ear can be approximated by a periodically repeated square pulse. Let $F(t)$ represent the gauge pressure at your ear. Take $F(t)$ to be $+1$ unit for the short time interval Δt, and zero before and after that interval. This "square pulse" of unit height and width Δt is repeated periodically at time intervals of length T. The short interval Δt gives the duration of each clapping sound. The period T is the time between successive claps. The frequency $\nu = \frac{1}{T}$ is the clapping frequency. You are to Fourier analyze $F(t)$.

(a) Show that you can choose the time origin so that only cosines of $n\omega t$ appear, i.e., so that

$$F(t) = B_0 + \sum_{n=1}^{\infty} B_n \cos(n\omega t) \tag{1}$$

(b) Show that $B_0 = \frac{\Delta t}{T}$, which is just the fractional "on" time. Show that

$$B_n = \frac{2}{n\pi} \sin(n\pi\nu\Delta t) \tag{2}$$

(c) Show that for $\Delta t << T$, the "fundamental" tone ν and the low harmonics 2ν, 3ν, 4ν, etc. all have essentially the same value for their Fourier amplitudes B_n.
(d) Sketch B_n versus $n\nu$, going up to sufficiently high n so that B_n has gone through zero two or three times.
(e) Show from part (d) that the "most important" frequencies (i.e., those with relatively large values for B_n) go from the fundamental, ν, to a frequency of the order of $\frac{1}{\Delta t}$. Thus we may call $\frac{1}{\Delta t}$ by the name ν_{max}. The "frequency band" of dominant frequencies has "bandwidth" equal to about $\nu_{max} = \frac{1}{\Delta t}$. The important frequencies are thus $\nu = 0, \nu, 2\nu, ...\nu_{max}$. The bandwidth of dominant frequencies can be given the name $\Delta\nu$. Then your result can be written $\Delta\nu\Delta t \approx 1$. This is a very important relation. It holds not only for our assumed $F(t)$, a repeated "square" pulse of width Δt, but for any pulse shape that can be characterized as being zero most of the time and nonzero for a time of duration about Δt.
(f) Using the equivalent form of the Fourier series $f(t) = \sum_{n=-\infty}^{\infty} C_n e^{in\omega t}$, find the amplitude C_n and plot it.

Problem 3

The pulses of Problem 2 now have amplitude $\frac{1}{\Delta t}$ with unit area under each pulse. Show that as Δt tends to zero, the infinite series of pulses is given by

$$f(t) = \frac{1}{T} + \frac{2}{T}\sum_{n=1}^{\infty} \cos \frac{n2\pi t}{T} \tag{3}$$

Under these conditions the amplitude of the original pulses becomes infinite, the energy per pulse remains finite and for an infinity of pulses in the train the total energy in the waveform is also infinite. The amplitude of the individual components in the frequency representation is finite, representing finite energy, but again, an infinity of components gives an infinite energy.