PHY201 Waves and Optics
Assignment 4

Handed out: September 02, 2018 Discussion: September 06, 2018

Problem 1

The coordinates of a particle of mass m are given by $(x, y) = (a\sin(\omega t), b\cos(\omega t))$ Eliminate t to show that the particle follows an elliptical path and show by adding its kinetic and potential energy at any position x, y that the ellipse is a path of constant energy equal to the sum of the separate energies of the harmonic vibrations. Prove that the quantity $m(x\dot{y} - y\dot{x})$ is also constant. What does this quantity represent?

Problem 2

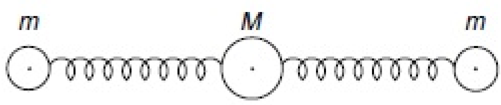
Two identical undamped oscillators, A and B, each of mass m and natural (angular) frequency ω_0, are coupled in such a way that the coupling force exerted on A is $\alpha m\frac{d^2x_B}{dt^2}$, and the coupling force exerted on B is $\alpha m\frac{d^2x_A}{dt^2}$, where α is a coupling constant of magnitude less than 1. Describe the normal modes of the coupled system and find their frequencies.

Problem 3

Consider a uniformly beaded string with N beads which has both the left end and the right end fixed. Assume that the tension in the string is T, the mass of each bead is m and the beads are spaced a distance a apart. Taking the trial solution for the normal mode as $y_i = A\sin(kx_i)\cos(\omega t - \phi)$, determine the dispersion relation $\omega(k)$.

Problem 4

The CO_2 molecule can be likened to a system made up of a central carbon atom with mass M connected by equal springs of spring constant k to two oxygen atoms of mass m. Set up and solve the equations for the two normal modes in which the masses oscillate along the line joining their centers. Describe the motion of the atoms in each mode.

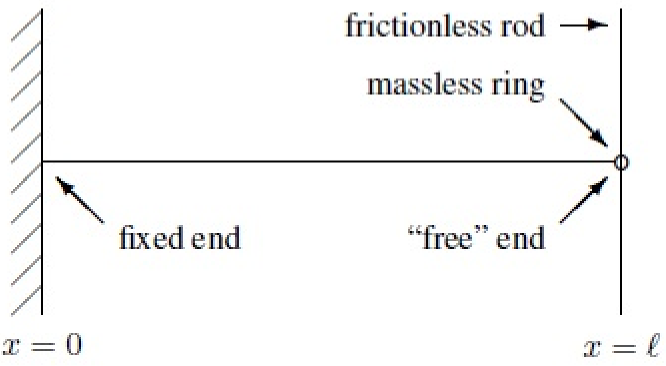


Problem 5

A string of length L and mass m, which is clamped at both ends and has a tension T, is pulled aside a distance h at its center and released.
(a) What is the energy of the subsequent oscillations? Hint: Consider the work done against the tension in giving the string its initial deformation.

(b) How often will the original shape reappear?
(c) Find the energy of the nth mode of oscillation.
(d) Find the total vibrational energy of all the modes of the string.

Problem 6



Consider a string with the $x = 0$ end fixed. Suppose that the other end, at $x = l$ is attached to a massless ring that is free to slide along a frictionless rod in the transverse direction, as shown in figure 1. We say that this system has one free end because the end at $x = l$ is free to slide in the transverse direction, even though it is fixed in the x direction. Because the rod is frictionless, the force on the ring due to the rod must have no component in the transverse direction. But because the ring is massless, the total force on the ring must vanish. Therefore, the force on the ring due to the string must have no component in the transverse direction. That implies that the string is horizontal at $x = l$. But the shape of the string at any given time is given by the graph of the transverse displacement, $y(x, t)$ versus x. Thus the slope of $y(x, t)$ at $x = l$ must vanish. Write down the appropriate boundary conditions for the displacement and determine the form of the normal mode solutions.

Problem 7

Prove each of the following statements using the "physical" method that makes use of the normal modes of a continuous string with appropriate boundary conditions.
(i) Any (reasonable) function $f(x)$ defined between $x = 0$ and $x = L$ and having value zero at $x = 0$ and slope zero at $x = L$ can be expanded in a Fourier series of the form

$$f(x) = \sum_n A_n \sin(nk_1x); \qquad n = 1, 3, 5, 7, ...; k_1L = \frac{\pi}{2}. \tag{1}$$

(ii) Any (reasonable) function $f(x)$ defined between $x = 0$ and $x = L$ and having zero slope at $x = 0$ and zero slope at $x = L$ can be expanded in a Fourier series of the form

$$f(x) = B_0 + \sum_n B_n \cos(nk_1x); \qquad n = 1, 2, 3, 4, ...; k_1L = \pi. \tag{2}$$

(iii) Any (reasonable) function $f(x)$ defined between $x = 0$ and $x = L$ and having zero slope at $x = 0$ and value zero at $x = L$ can be expanded in a Fourier series of the form

$$f(x) = \sum_n B_n \cos(nk_1x); \qquad n = 1, 3, 5, 7, ...; k_1L = \frac{\pi}{2}. \tag{3}$$