PHY201 Waves and Optics
Assignment 2

Handed out: August 9, 2018 Discussion: August 16, 2018

Problem 1

A block of mas m is connected to a spring, the other end of which is fixed. There is also a viscous damping mechanism. The following observations have been made on this system:
(1) If the block is pushed horizontally with a force equal to mg, the static compression of the spring is equal to h.
(2) The viscous resistive force is equal to mg if the block moves with a certain known speed u.
(a) For this complete system (including both spring and damper), write the differential equation governing horizontal oscillations of the mass in terms of m, g, h and u.
Answer the following for the case that $u = 3\sqrt{gh}$:
(b) What is the angular frequency of the damped oscillations?
(c) After what time, expressed as a multiple of $\sqrt{\frac{h}{g}}$, is the energy down by a factor $\frac{1}{e}$?
(d) What is the Q of this oscillator?
(e) This oscillator, initially in its rest position, is suddenly set into motion at $t = 0$ by a bullet of negligible mass but non-negligible momentum travelling in the positive x direction. Find the value of the phase angle δ in the equation $x = Ae^{-\frac{\gamma}{2}t}\cos(\omega t - \delta)$ that describes the subsequent motion, and sketch x versus t for the first few cycles.
(f) If the oscillator is driven with a force $mg\cos(\omega t)$, where $\omega = \sqrt{\frac{2g}{h}}$, what is the amplitude of the steady-state response?

Problem 2

A mass m is subject to a resistive force $-bv$ but no springlike restoring force.
(a) Show that its displacement as a function of time is of the form $x = C - \frac{\nu_0}{\gamma}e^{-\gamma t}$ where $\gamma = \frac{b}{m}$.
(b) At $t = 0$, the mass is at rest at $x = 0$. At this instant, a driving force $F = F_0 \cos\omega t$ is switched on. Find the values of A and δ in the steady-state solution $x = A\cos(\omega t - \delta)$.
(c) Write down the general solution (the sum of parts (a) and (b)) and find the values of C and ν_0 from the conditions that $x = 0$ and $\frac{dx}{dt} = 0$ at $t = 0$. Sketch x as a function of t.

Problem 3

A forced damped oscillator of mass m has a displacement varying with time given by $x = A\sin(\omega t)$. The resistive force is $-b\nu$. From this information, calculate how much work is done against the resistive force during one cycle of oscillation.

Problem 4

The equation $\ddot{x} + \omega_0^2 x = -\frac{eE_0}{m}\cos\omega t$ describes the motion of a bound undamped electric charge $-e$ of mass m under the influence of an alternating electric field $E = E_0 \cos(\omega t)$. For an electron number density n, show that the induced polarizability χ per unit volume (the dynamic susceptibility) of a medium is

$$\chi_e = -\frac{nex}{\epsilon_0 E} = \frac{ne^2}{\epsilon_0 m(\omega_0^2 - \omega^2)} \tag{1}$$

The permittivity of a medium is defined as $\epsilon = \epsilon_0(1+\chi)$ where ϵ_0 is the permittivity of free space. The relative permittivity $\frac{\epsilon}{\epsilon_0}$ is called the dielectric constant.

Problem 5

(a) The equation of motion for a damped one-dimensional harmonic oscillator is

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \tag{2}$$

For given initial conditions $x(0)$ and $\dot{x}(0)$, show that the solution $x(t)$ can be written in the form

$$x(t) = e^{-\frac{\gamma}{2}t}\left[x(0)\cos(\omega_1 t) + \left(\dot{x}(0) + \frac{\gamma}{2}x(0)\right)\frac{\sin(\omega_1 t)}{\omega_1}\right] \tag{3}$$

where $\omega_1 = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}$.

(b) Show that if $x_1(t)$ and $x_2(t)$ are solutions to Eq. (2), then the solution $x(t) = x_1(t) + x_2(t)$ is also a solution provided that the initial conditions $x(0)$ and $\dot{x}(0)$ for the superposition are also the corresponding sums of the initial conditions, i.e., provided $x(0) = x_1(0) + x_2(0)$ and $\dot{x}(0) = \dot{x}_1(0) + \dot{x}_2(0)$.