MTH 202: Probability and Statistics
HW 4 (due date: 09/02/2015)

Problems for submission

1. Let X be a geometrically distributed random variable with parameter p. Let $M > 0$ be an integer. Let $Z = \min(X, M)$. Compute the mean of Z.

2. Suppose a box has 5 balls labeled $1, 2, 3, 4$, and 5. Two balls are selected without replacement from the box. Let X denote the number on the first ball and Y denote the number on the second ball. Compute $\text{Cov}(X, Y)$ and $\rho(X, Y)$.

3. Let X and Y be two independent Poisson distributed random variables with parameters λ_1 and λ_2, respectively. Compute $E[Y|X + Y = z]$ where z is a non-negative integer.

4. A coin is tossed until a head or four tails in succession occurs. Let X denote the number of tosses required. Compute $E[X]$.

5. Suppose the probability of a tennis player winning a match against his opponent in a tour is 0.6. The tour consists of 10 matches. Let X denote the number of matches this player will win before his first defeat occurs. Find the density function of X. Find the mean and variance of X.

Problems not for submission

1. Let X be a discrete random variable. Compute the value of a for which $E(X - a)^2$ is minimum.

2. Let X be a random variable having a negative binomial distribution with parameters α and p. Find the mean and variance of X.

3. Let X be a geometrically distributed random variable with parameter p. Let $M > 0$ be an integer. Let $Z = \max(X, M)$. Compute the mean of Z.

4. Construct an example of a density function which has a finite moment of order 4, but has infinite moments of order 5 and above.

5. Suppose X and Y are independent random variables having finite moments of order 2. Compute the mean and variance of $5X - 4Y$ in terms of mean and variance of X and Y.

6. Let X and Y be two independent geometrically distributed random variables with parameter p. Compute $E[Y|X + Y = z]$ where z is a non-negative integer.

7. Suppose X and Y are independent random variables such that $EX^4 = 2$, $EY^2 = 1$, $EX^2 = 1$, and $EY = 0$. Compute the variance of X^2Y.