MTH 202: Probability and Statistics
HW 3 (due date: 02/02/2015)

Problems for submission

1. Let α be a positive real number and $0 < p < 1$. The negative binomial density with parameters α and p is defined as $f(x) = p^\alpha \binom{-\alpha}{x} (-1)^x (1-p)^x$, $x = 0, 1, 2, \cdots$; and $f(x) = 0$, otherwise. Here $\binom{-\alpha}{x} = \frac{-\alpha(-\alpha-1)\cdots(\alpha-x+1)}{x!}$.
   Let X be a random variable with negative binomial density with parameters $\alpha = n$ (where, n is a fixed natural number) and p. Compute the density function of $X + r$.

2. Let X be a random variable taking finitely many values $x_1, \cdots, x_n$. X is said to be uniformly distributed if $P(X = x_i) = 1/n$, for $i = 1, \cdots, n$.
   Let X be a uniformly distributed random variable taking values $0, 1, \cdots, 200$.
   (a) Compute the distribution function of X.
   (b) Compute $P(101 < X \leq 200)$.

3. Let X be geometrically distributed with parameter p. Compute the density of
   (a) X^3.
   (b) $X + 4$.

4. Let X, Y, and Z be independent random variables having Poisson densities with parameters λ_1, λ_2, and λ_3, respectively. For non-negative integers, x, y, and z, compute
$$P(X = x, Y = y, Z = z \mid X + Y + Z = x + y + z).$$

5. Suppose in a group of people, 5% have invalid driver's license. Use the Poisson approximation to calculate the probability that at most 2 out of 50 given people will have invalid driver's licenses.

Problems not for submission

1. A die is rolled until a 6 appears.
   (a) What is the probability that at most six rolls are needed?
   (b) How many rolls are required so that the probability of getting 6 is at least 1/2?

2. Let $X_1, \cdots, X_r$ be discrete random variables on a probability space $(\Omega, \mathcal{A}, P)$. The joint density function of $X_1, \cdots, X_r$ is defined as $f(x_1, \cdots, x_r) = P(X_1 = x_1, \cdots, X_r = x_r)$.
   Suppose $2r$ balls are distributed at random into r boxes. Let X_i denote the number of balls in box i.
   (a) Find the joint density of $X_1, \cdots, X_r$.
   (b) Find the probability that each box contains exactly 2 balls.

3. Let X and Y be two uniformly distributed random variables on the set $0, 1, \cdots, N$. Compute the density of $X + Y$.

4. There are N tickets numbered $1, 2, \cdots, N$ from which n tickets are chosen. Let X denotes the smallest numbers on the n tickets drawn. Find the density and distribution function of X.

5. Give an example of two random variables X and Y that are not independent, but the random variables X^2 and Y^2 are independent.