

MTH 202: Probability and Statistics  
HW 1 (due date: 13/01/2015)

**Problems for submission**

1. A box contains 10 balls, numbered 1 through 10. A ball is drawn from the box at random and then a ball is drawn at random from the remaining 9 balls. Compute the probability that the number on the two drawn balls is atleast 2.
2. Suppose two six-faced dice are rolled once and all the possible outcomes are equally likely. Find the probability that the sum of the numbers on the two faces is odd.
3. A student is taking a multiple choice exam in which each question has 4 possible answers, exactly one of which is correct. If the student knows the correct answer, he selects the correct answer. Otherwise, he selects one answer at random from the possible 4 choices. Suppose the student knows the answer to 70% of the questions.
  - (a) What is the probability that on a given question, the student gets the correct answer?
  - (b) If the student gets the correct answer to a question, what is the probability that he knows the answer?
4. Suppose the probability of hitting a target is  $1/4$ . If eight shots are fixed at the target, what is the probability that the target is hit atleast twice?
5. Let  $A, B$  and  $C$  be three events on a sample space  $(\Omega, \mathcal{A})$  (where,  $\mathcal{A}$  is a  $\sigma$ -field of  $\Omega$ ) such that  $A \cup B \cup C = \Omega$ .
  - (a) What is the event that only  $A$  occurs?
  - (b) What is the event that at least two of  $A, B, C$  occur?
  - (c) What is the event that both  $A$  and  $C$  occur, but  $B$  does not occur?
  - (d) What is the event that atleast one of  $A, B, C$  occur?

**Problems not for submission**

1. Let  $A$  and  $B$  denote two independent events with sample space  $\Omega$ . Show that  $A^c$  and  $B$ ;  $A$  and  $B^c$ ; and  $A^c$  and  $B^c$  are also independent.
2. Show that if  $A, B$ , and  $C$  are events with  $P(A \cap B \cap C) \neq 0$  and  $P(C|A \cap B) = P(C|B)$ , then  $P(A|B \cap C) = P(A|B)$ .
3. Two real numbers  $r < s$  are chosen at random in the interval  $[0, 1]$ . What is the probability that the lengths of all the line segments  $[0, r], [r, s], [s, 1]$  have length atleast  $1/4$ ?
4. A box contains 10 balls, numbered 1 through 10. A ball is drawn from the box at random. Compute the probability that the number on the ball is either 3, 4, or 10.
5. Let  $A$  and  $B$  be two events such that  $A \subseteq B$ . What is
  - (a)  $P(A \cup B)$ , (b)  $P(A \cap B)$ , and (c)  $P(A \setminus B)$ ?