This statement is a short summary of the research work I have done so far. I will mention the results that I have proved, and also briefly explain the ideas and techniques which are used.

1. Self-dual representations and Signs

Let $F$ be a non-Archimedean local field and $G$ be the group of $F$-points of a connected reductive algebraic group. Let $(\pi, W)$ be a smooth irreducible complex representation of $G$. We write $(\pi^\vee, W^\vee)$ for the smooth dual or contragredient of $(\pi, W)$ and $\langle , \rangle$ for the canonical non-degenerate $G$-invariant pairing on $W \times W^\vee$ (given by evaluation). Let $\theta$ be a continuous automorphism of $G$ of order at most two. Let $(\pi^\theta, W)$ be the $\theta$-twist of $\pi$ defined by

$$\pi^\theta(g)w = \pi(\theta(g))w.$$ 

Suppose that $\pi^\theta \simeq \pi^\vee$. In the presence of Schur’s lemma, it can be shown that there exists a non-degenerate bilinear form $( ,)$ on $W$ satisfying the following invariance property

$$\langle \pi(g)w_1, \pi^\theta(g)w_2 \rangle = \langle w_1, w_2 \rangle, \quad \forall w_1, w_2 \in W.$$ 

Using a simple application of Schur’s lemma, it can be shown that this form is unique up to scalars, and consequently is either symmetric or skew-symmetric. Accordingly, we set

$$\varepsilon_\theta(\pi) = \begin{cases} 1 & \text{if the form is symmetric}, \\ -1 & \text{if the form is skew-symmetric}, \end{cases}$$

and call it the twisted sign of $\pi$. It clearly depends only on the equivalence class of $\pi$. If $\theta$ is the trivial automorphism of $G$, we simply write $\varepsilon(\pi)$ instead of $\varepsilon_1(\pi)$ and call it the ordinary sign.

1.1. Self-dual representations with vectors fixed under an Iwahori subgroup. Let $G$ be the group of $F$-points of a split connected reductive $F$-group over a non-Archimedean local field $F$ of characteristic 0 (for eg. $G = \text{GL}(n, F)$). Suppose that $(\pi, V)$ is a complex irreducible smooth self-dual representation of $G$. In this situation, the sign $\varepsilon(\pi)$ is well defined. In this paper, we proved that $\varepsilon(\pi) = 1$ when $\pi$ is a “generic” representation of $G$ with non-trivial vectors fixed under an Iwahori subgroup.

Before we state the main theorem, we set up some notation and recall the definitions of an Iwahori subgroup and generic representations. To keep the exposition simple, we restrict ourselves to the group $\text{GL}(n, F)$.

Let $F$ be a non-Archimedean local field. We write $\mathfrak{o}$ for the ring of integers in $F$, $\mathfrak{p}$ for the unique maximal ideal in $\mathfrak{o}$ and $k = \mathfrak{o}/\mathfrak{p}$ for the finite residue field of order $q$. We also fix a Borel subgroup $B$ defined over $F$. We write $U$ for the unipotent radical of $B$.

**Definition 1.1.** The Iwahori subgroup $I$ is defined to be the inverse image of $B(k)$ ($k$-points of $B$) under the canonical map (reduction mod $\mathfrak{p}$) $G(\mathfrak{o}) \to G(k)$.

**Example 1.2.** Let $G = \text{GL}(n, F)$. Take $B$ to be the standard Borel subgroup (upper triangular matrices) and $U$ to be the unipotent radical (unipotent matrices) of $B$ in $G$. In
this case the Iwahori subgroup is the collection of matrices of the following type:

$$ I = \begin{pmatrix} \mathfrak{o} & \mathfrak{o} & \cdots & \mathfrak{o} \\ p & \mathfrak{o} & \cdots & \mathfrak{o} \\ \vdots & \vdots & \ddots & \vdots \\ p & p & \cdots & \mathfrak{o} \end{pmatrix} $$

**Definition 1.3.** A representation $\pi$ is called generic if there exists a “non-degenerate” character $\psi$ of $U$ such that $\text{Hom}_U(\pi, \psi) \neq 0$.

**Remark 1.4.** When $G = \text{GL}(n, F)$, any non-degenerate character $\psi$ of $U$ is given by

$$ \psi(u) = \theta(\alpha_1 u_{12} + \alpha_2 u_{23} + \cdots + \alpha_{n-1} u_{n-1n}) $$

where $\theta$ is a complex valued non-trivial additive character of $F$, $u = (u_{ij})$ and $\alpha_1, \ldots, \alpha_n \in F^\times$.

We will now briefly explain the main ideas of the proof. In [15], Prasad gives a criterion to compute the sign for an irreducible smooth self-dual generic representation of a $p$-adic group $G$. Assuming the existence of a “special” element $s \in T$ (where $T$ is a maximal $F$-split torus), he shows that $s^2 \in Z$ is a central element in $G$ and in the case when $\pi$ is generic, the sign is determined by evaluating the central character at $s^2$. We have used this trick of Prasad to compute the sign.

To prove our main result, we consider the cases when $G$ has either a connected center or disconnected center. In the situation when $Z$ is connected, we prove the existence of the special element $s \in T$. In fact, we show that $s \in T(\mathfrak{o})$. Using the fact that $\pi^T \neq 0$, it is immediate that $\varepsilon(\pi) = 1$ in this case. When $Z$ is disconnected, we construct a split connected reductive $F$-group $\tilde{G}$ with a maximal $F$-split torus $\tilde{T}$ so that $G$ imbeds as a subgroup of $\tilde{G}$. This group $\tilde{G}$ has a connected center $\tilde{Z}$. We show that there exists a self-dual character $\nu$ of $\tilde{Z}$ extending the central character $\omega_{\pi}$. Using this character $\nu$, we extend $\pi$ to a representation of $\tilde{Z}G$. We show that the quotient group $\tilde{G}/(\tilde{Z}G)$ is a finite abelian group. We use the results of Gelbart and Knapp (see lemma 2.1 and lemma 2.3 in [10]) to get an irreducible representation $\tilde{\pi}$ of $\tilde{G}$ containing $\pi$. We prove that $\tilde{\pi}$ can be chosen to have non-zero vectors fixed under an Iwahori subgroup in $\tilde{G}$. We show that the representation $\tilde{\pi}$ is generic and hence it contains $\pi$ with multiplicity one on restriction to $G$. Also, $\tilde{\pi}$ is self-dual up to a twist by a character $\chi$ of $\tilde{G}$. Since center of $\tilde{G}$ is connected we get an element $s \in T(\mathfrak{o})$ satisfying the hypotheses of Prasad’s Theorem. We prove that $\varepsilon(\tilde{\pi}) = \chi(s)\omega_{\tilde{\pi}}(s^2)$ and $\varepsilon(\tilde{\pi}) = \varepsilon(\pi)$. Finally using the fact that $\tilde{\pi}$ has Iwahori fixed vectors we show that $\chi(s) = \omega_{\tilde{\pi}}(s^2) = 1$.

We now state the main result proved in this paper. For more details about the work, we refer the reader to [2].

**Theorem 1.5.** Let $(\pi, V)$ be an irreducible smooth self-dual representation of $G$ with non-zero vectors fixed under an Iwahori subgroup in $G$. Suppose that $\pi$ is also generic. Then $\varepsilon(\pi) = 1$. 

1.2. Self-dual representations of $\text{SL}(n,F)$. Let $F$ be a non-Archimedean local field of characteristic 0 and $G = \text{SL}(n,F)$. Suppose that $(\pi,V)$ is a complex irreducible smooth self-dual representation of $G$ with non-trivial vectors fixed under an Iwahori subgroup. If we know that $(\pi,V)$ is a generic representation, we can use our previous work (see section 1.1 above) and compute the sign. In this paper, we extended our results to include the case of non-generic representations of $G$. To be more precise, we showed that for any representation $(\pi,V)$ of $G$ as above, we have $\varepsilon(\pi) = 1$.

Since $(\pi,V)$ is a non-generic representation, we cannot apply the ideas of Prasad (which work only for generic representations) to compute the sign. The key idea in this work was to use the results of Roche and Spallone (see [16]) and simplify the problem to computing the twisted sign of a certain generic representation of a Levi subgroup of $G$.

For the sake of clarity, we briefly recall their method below. We continue with the same notation and terminology as in [16]. For further details, we refer the reader to sections §3, §4 of [16].

Let $\theta$ be an involutory automorphism of $G$ and suppose that $\pi^\theta \simeq \pi^\vee$. Let $(P,\tau,\nu)$ be the triple associated to $\pi$ via the Langlands’ classification. Suppose that $P$ has Levi decomposition $P = MN$. Under certain assumptions on the involution $\theta$, they apply Casselman’s pairing to show that $\varepsilon_\theta(\pi) = \varepsilon_\theta(\pi_N)$, where $\pi_N$ is the Jacquet module of $\pi$. Using $\pi^\theta \simeq \pi^\vee$ and the fact that $\tau$ occurs with multiplicity one as a composition factor of $\pi_N$, they prove the following:

**Theorem 1.6 (Roche-Spallone).** Let $\pi$ be an irreducible smooth representation of $G$ such that $\pi^\theta \simeq \pi^\vee$. Suppose the Langlands’ classification attaches the triple $(P,\tau,\nu)$ to $\pi$. Then $\tau^\theta \simeq \tau^\vee$ and $\varepsilon_\theta(\pi) = \varepsilon_\theta(\tau)$.

In our work, we explicitly constructed an involution $\theta$ of $G$ such that the representation $\pi^\theta \simeq \pi^\vee$. We also showed that $\varepsilon_\theta(\pi) = \varepsilon(\pi)$ and hence it is enough to determine the twisted sign. The involution $\theta$ satisfies all the hypotheses needed to apply the Casselman’s pairing. Using the results of Roche and Spallone, we can further reduce to the case of computing the twisted sign of a tempered representation $\tau$ of a certain Levi subgroup of $G$. In the case when $G = \text{SL}(n,F)$, we show that the tempered representation $\tau$ is also generic. Once we have reduced to this stage, we employ the methods of the previous work (see section 1.1) to compute the sign.

We now state the main result proved in this paper. For more details about the work, we refer the reader to [3].

**Theorem 1.7.** Let $(\pi,V)$ be an irreducible smooth self-dual representation of $G$ with non-zero vectors fixed under an Iwahori subgroup in $G$. Then $\varepsilon(\pi) = 1$.

1.3. Self-dual representations of $\text{Sp}(4,F)$. Let $F$ be a non-Archimedean local field of characteristic 0 and $G = \text{Sp}(4,F)$. Let $(\pi,W)$ be an irreducible smooth self-dual representation of $G$. In this paper, we study the sign $\varepsilon(\pi)$ and we show that $\varepsilon(\pi) = 1$ when $\pi$ is an Iwahori-spherical representation of $G$. 

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The representation theory of p-adic symplectic group is quite involved and difficult to understand (see for example [18], [19], [20], [21]). The primary reason to attempt a particular case was to understand the problem in this context and hopefully use it to study the general case. Unfortunately, we have had to restrict ourselves only to this particular case, since our method uses a result about the similitude group GSp(n, F) which is not valid for n > 4. To be more precise, we have used the fact that square-integrable, Iwahori-spherical representations of GSp(4, F) are generic.

We now briefly summarize the key ideas of the proof. We first consider the case when \( \pi \) is a square-integrable (discrete series) representation of \( G \). We prove that such representations are generic. When \( \pi \) is a generic representation of \( G \), it is well known from the work of Prasad (proposition 2 in [14]) that the sign is given by \( \omega_\pi(-1) \), where \( \omega_\pi \) is the central character of \( \pi \). Using this along with the fact that \( \pi \) is Iwahori-spherical, it follows immediately that \( \varepsilon(\pi) = 1 \).

In the case when \( \pi \) is not generic, we consider the cases when \( \pi \) is not tempered and \( \pi \) is tempered.

We first explain the key ideas in the case when \( \pi \) is not tempered. Let \( w_0 \in \text{GL}(4, F) \) to be the matrix with anti-diagonal entries 1 and let \( \theta(g) = w_0gw_0^{-1} \). Using a result of Waldspurger (See Chapter 4.II.1 in [13]), it follows that \( \pi^\theta \simeq \pi^\vee \). For \( g \in \tilde{G} = \text{GSp}(4, F) \), we let \( \iota(g) = \lambda_\theta^{-1}w_0gw_0^{-1} \), where \( \lambda_\theta \) is the multiplier of \( g \). It is easy to see that \( \iota \) is a continuous automorphism of \( \tilde{G} \) of order two and the restriction of \( \iota \) to \( G \) is \( \theta \). Let \( \tilde{\pi} \) be an irreducible smooth representation of \( \tilde{G} \) such that the restriction of \( \tilde{\pi} \) to \( G \) contains the representation \( \pi \) with multiplicity one (see [1]). It can be shown that \( \tilde{\pi}^\iota \simeq \tilde{\pi}^\vee \) (we refer the reader to Theorem B in [17] for a proof) and thus \( \varepsilon\iota(\tilde{\pi}) \) makes sense. Since \( (\iota|_G = \theta \), and the restriction of the bilinear form on \( \tilde{\pi} \) satisfying
\[
[\tilde{\pi}(g)(v), \tilde{\pi}(g)(w)] = [v, w]
\]
to \( \pi \) is non-degenerate and hence we have
\[
\varepsilon\iota(\tilde{\pi}) = \varepsilon\theta(\pi).
\]

Consider the representation \( \tilde{\pi}^\vee \simeq \tilde{\pi} \otimes \omega_{\tilde{\pi}}^{-1} \) (Here \( \omega_{\tilde{\pi}} \) is the character obtained by composing the central character \( \omega_\tilde{\pi} \) with the map \( \lambda: \tilde{G} \rightarrow F^\times \)). Since \( \tilde{\pi} \simeq \pi^\vee \), it follows that the restriction of \( \tilde{\pi}^\vee \) to \( G \) also contains \( \pi \) with multiplicity one. The bilinear form \( (v, w) \) on \( \tilde{\pi} \) satisfying
\[
(\tilde{\pi}(g)v, \tilde{\pi}(g)w) = \omega_{\tilde{\pi}}(\lambda_\theta)(v, w)
\]
restricts to a non-degenerate \( G \)-invariant bilinear form on \( \pi \) and it follows that
\[
\varepsilon(\tilde{\pi}) = \varepsilon(\pi).
\]

We use the above results to establish a relation between the twisted sign \( \varepsilon\theta(\pi) \) and the ordinary sign \( \varepsilon(\pi) \). To be more precise, we show that
\[
\varepsilon\theta(\pi) = \varepsilon(\pi)\omega_\pi(-1).
\]

Using the fact that \( \pi \) has non-zero Iwahori fixed vectors, it follows that \( \varepsilon\theta(\pi) = \varepsilon(\pi) \) and hence it is enough to determine that twisted sign \( \varepsilon\theta(\pi) \). We use the results of Roche and Spallone ([16]) and further reduce the problem to computing the twisted sign of a certain tempered representation of a Levi subgroup \( M \) of \( G \). In our case we have the following possibilities for \( M \):
(a) \( M \simeq \text{GL}(1, F) \times \text{GL}(1, F) \),
(b) \( M \simeq \text{GL}(2, F) \),
(c) \( M \simeq \text{SL}(2, F) \times \text{GL}(1, F) \).

Using the fact that the Levi subgroup of \( G \) is one of the above types, allows us to apply some known results on Iwahori-spherical representations to compute the sign.

In the case when \( \pi \) is tempered, we compute the sign by reducing to the case of studying the twisted sign of a certain discrete series representation of a Levi subgroup of \( G \). For the sake of clarity, we summarize the important ideas and results which go into proving this case.

Since \( \pi \) is a tempered representation, it follows from a result of Harish-Chandra (see III.4.1 in [22]), that there exists a (proper) parabolic subgroup \( P = MN \) and an irreducible smooth discrete series representation \( \sigma \) of \( M \) such that \( \pi \hookrightarrow \text{ind}^G_P \sigma \). Since \( \sigma \) is a discrete series representation, \( \pi \) occurs with multiplicity one in \( \text{ind}^G_P \sigma \) (see [11], [12]). As earlier, we consider the different possibilities for \( M \) and in all these cases, we show that \( \sigma^\theta \simeq \sigma^\vee \) and thus \( \varepsilon_\theta(\pi) \) is well defined. The next key step in the proof is to relate the signs \( \varepsilon_\theta(\pi) \) and \( \varepsilon_\theta(\sigma) \). To be more precise, we show that

\[
\varepsilon_\theta(\pi) = \varepsilon_\theta(\sigma).
\]

To establish the above equality of signs, we separate the cases when \( \sigma \) is regular (for \( n \in N_G(M) \), if \( \sigma^n \simeq \sigma \), then \( n \in M \) ) or not regular.

When \( \sigma \) is regular, \( \text{ind}^G_P \sigma \) is irreducible (see Corollary 1.2 in [11]). Therefore, we have \( \pi = \text{ind}^G_P \sigma \). Also it can be shown that \( \pi_N = (\text{ind}^G_P \sigma)_N \) contains \( \sigma \) with multiplicity one (see III.7.3 in [22]). From this it follows that \( \varepsilon_\theta(\pi) = \varepsilon_\theta(\sigma) \). When \( \sigma \) is not regular, \( \text{ind}^G_P \sigma \) may not be an irreducible representation. This adds more complexity to establishing the above equality of the signs. We refer the reader to section 4.3.2 in [4] for the details of this computation.

Since \( \sigma \) is an irreducible discrete series representation of \( M \), it follows that \( \sigma \) is generic. Proceeding as in the previous case and using the fact that \( \sigma \) is Iwahori spherical, we can see that \( \varepsilon_\theta(\sigma) = 1 \).

We now state the main result proved in this paper. For more details about the work, we refer the reader to [4].

**Theorem 1.8.** Let \( (\pi, V) \) be an irreducible smooth self-dual representation of \( G \) with non-zero vectors fixed under an Iwahori subgroup in \( G \). Then \( \varepsilon(\pi) = 1 \).

1.4. **Self-dual representations of SL(2, F) - an approach using the Iwahori-Hecke algebra.** Let \( G = \text{SL}(2, F) \) and \( (\pi, V) \) be a complex, irreducible, smooth, Iwahori-spherical representation of \( G \). It can be shown that such representations are always self-dual and hence the sign \( \varepsilon(\pi) \) makes sense. In this paper, we reproved a particular case of an earlier result of mine (see section 1.2 above) using a completely different approach. This is a joint work with K.S. Senthil Raani and Brahadeesh Sankarnarayanan.

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Before we state our result, we will set up a few preliminaries and explain the novelty of this approach and the motivation to attempt a different proof of the problem.

Let \( G = \text{SL}(2, F) \) and \( I \) be an Iwahori subgroup in \( G \). Let \( \mu \) be a Haar measure on \( G \). We normalize the Haar measure \( \mu \) such that \( \mu(I) = 1 \). Let \( \mathcal{H}(G, I) \) be the space of all functions \( f : G \to \mathbb{C} \) which are locally constant, compactly supported and \( I \) bi-invariant. It is well known that \( \mathcal{H}(G, I) \) is an associative \( \mathbb{C} \)-algebra with respect to usual convolution of functions. It is called the Iwahori-Hecke algebra of \( G \). Let \( M = V^I \) be the set of all vectors in \( V \), which is fixed under the Iwahori subgroup. It is well known that \( M \) becomes a simple module over \( \mathcal{H}(G, I) \) (see section 4 in [7]) via, the action

\[
\mathcal{H}(G, I) \times M \to M : (f, v) \mapsto f \cdot v
\]

where

\[
f \cdot v = \pi(f)(v) = \int_G f(g)\pi(g)(v)d\mu(g).
\]

The algebra \( \mathcal{H}(G, I) \) has a well known presentation due to Bernstein and Lusztig. For the sake of simplicity, we only explain it for the Iwahori-Hecke algebra of \( \text{SL}(2, F) \). We refer the reader to [8] (see chapter 3, section 1), for more details about the presentation in a very general setup.

Throughout, we write \( \mathfrak{o} \) for the ring of integers in \( F \), \( \mathfrak{p} \) for the unique maximal ideal in \( \mathfrak{o} \) with generator \( \varpi \) and \( k \) for the finite residue field of cardinality \( q \). We let \( T \) denote the subgroup of diagonal matrices in \( G \), and let \( T_\varnothing = T \cap I \). We write \( W = N_G(T)/T \) for the (finite) Weyl group, \( \widetilde{W} = N_G(T)/T_\varnothing \) for the (infinite) affine Weyl group. Let

\[
s_0 = \begin{bmatrix} 0 & \varpi \\ -\varpi & 0 \end{bmatrix}, \quad s_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} \varpi & 0 \\ 0 & \varpi & -1 \end{bmatrix}.
\]

It can be shown that \( \widetilde{W} = \langle s_0, s_1 | s_0^2 = 1, s_1^2 = 1 \rangle \). We let \( R = \mathbb{C}[q^{1/2}, q^{-1/2}] \). For \( L \subset G \), we write \( \chi_L \) for the characteristic function of \( L \). Let \( \theta = q^{-1}\chi_{IwI} \). It can be shown that \( \theta \) is an invertible element in \( \mathcal{H} \) and \( A = \text{Span}_R \{ \theta^n | n \in \mathbb{Z} \} \) is an abelian subalgebra of \( \mathcal{H} \). For \( w \in \widetilde{W} \), we let \( N_w = q^{-1/2}\chi_{IwI} \).

**Proposition 1.9** (Bernstein-Lusztig). Let \( s = s_1 \) and \( \mathfrak{B} = \{ \theta^n, N_s\theta^n | n \in \mathbb{Z} \} \). Then \( \mathfrak{B} \) is an \( R \)-basis for \( \mathcal{H} \) and \( \mathcal{H} \) is generated as an algebra subject to the following relations:

\[
a) \ (N_s - q^{1/2})(N_s + q^{-1/2}) = 0.
\]

\[
b) \ \theta N_s - N_s\theta^{-1} = \beta(\theta + 1).
\]

The key idea of our paper was to relate the sign \( \varepsilon(\pi) \) of the representation to the sign \( \varepsilon(M) \) of the simple module \( M \). To be more precise, we proved that \( \varepsilon(\pi) = \varepsilon(M) \). It is well known that \( \dim_{\mathbb{C}} M \leq |W| \). In our case \( W \) happens to be a group of order 2. The was crucial and we were able to determine \( \varepsilon(M) \) using just some elementary linear algebra.

The main motivation behind this work is to learn about the Iwahori-Hecke algebra and the Bernstein-Lusztig presentation and ultimately use it to simplify the earlier proof for the \( \text{SL}(n, F) \) case. The previous proof for \( \text{SL}(n, F) \) involved a lot of technical details and also works only when the characteristic of \( F \) is zero. We have observed that the proof using the
Bernstein-Lusztig presentation of the Iwahori-Hecke algebra greatly simplifies the technical details and also doesn’t have any dependence on the characteristic of the field $F$.

We now state the main result proved in this paper. For more details about the work, the interested reader is referred to (add reference).

**Theorem 1.10.** Let $(\pi, V)$ be an irreducible smooth representation of $\text{SL}(2, F)$ with non-zero vectors fixed under an Iwahori subgroup. Then $\varepsilon(\pi) = 1$.

2. Finite order elements in the integral Symplectic group

Given a group $G$ and a positive integer $m \in \mathbb{N}$, it is natural to ask if there exists $k \in G$ such that $o(k) = m$, where $o(k)$ denotes the order of the element $k$. In the case when $G = \text{Sp}(2g, \mathbb{Z})$, there is a classification of finite order elements in $G$ by Bürgisser (see corollary 2 in [6]). We mention his result for the sake of clarity.

**Proposition 2.1** (Bürgisser). Let $m = p_1^{\alpha_1} \cdots p_k^{\alpha_k}$, where the primes $p_i$ satisfy $p_i < p_{i+1}$ for $1 \leq i < k$ and where $\alpha_i \geq 1$ for $1 \leq i \leq k$. There exists a matrix $A \in \text{Sp}(2g, \mathbb{Z})$ of order $m$ if and only if

\begin{equation}
\begin{align*}
a) \sum_{i=1}^{k} \varphi(p_i^{\alpha_i}) &\leq 2g, \text{ if } m \equiv 2(\text{mod } 4). \\
b) \sum_{i=1}^{k} \varphi(p_i^{\alpha_i}) &\leq 2g, \text{ if } m \not\equiv 2(\text{mod } 4).
\end{align*}
\end{equation}

For $g \in \mathbb{N}$, let $G = \text{Sp}(2g, \mathbb{Z})$ be the integral symplectic group and $S(g)$ be the set of all positive integers which can occur as the order of an element in $G$. Using the above result of Bürgisser, it is not too difficult to see that $S(g)$ is a bounded subset of $\mathbb{R}$. Hence it makes sense to consider to functions $f(g) = |S(g)|$, and $h(g) = \max\{m \in \mathbb{N} \mid m \in S(g)\}$. In a joint work with Ram Murty and Karam Deo Shankhadhar, we have studied the growth of the functions $f$ and $h$ and show that they have at least exponential growth.

The above question derives its motivation from analogous questions from the theory of mapping class groups of a surface of genus $g$ (see section 2.1 in [9] for the definition). We know that given a closed oriented surface $S_g$ of genus $g$, there is a surjective homomorphism $\psi : \text{Mod}(S_g) \to \text{Sp}(2g, \mathbb{Z})$, where $\text{Mod}(S_g)$ is the mapping class group of $S_g$ (see theorem 6.4 in [9]). It is a well known fact that for $f \in \text{Mod}(S_g)$ ($f \neq 1$) of finite order, we have $\psi(f) \neq 1$. Let $\tilde{S}(g) = \{m \in \mathbb{N} \mid \exists f \neq 1 \in \text{Mod}(S_g) \text{ with } o(f) = m\}$. The set $\tilde{S}(g)$ is a finite set and it makes sense to consider the functions $\tilde{f}(g) = |\tilde{S}(g)|$ and $\tilde{h}(g) = \max\{m \in \mathbb{N} \mid m \in \tilde{S}(g)\}$. It is a well known fact that both these functions $\tilde{f}$ and $\tilde{h}$ are bounded above by $4g + 2$ (see corollary 7.6 in [9]).

We now state the main result proved in this paper. For more details, we refer the interested reader to [5].

**Theorem 2.2.** For $g \in \mathbb{N}$, let $G = \text{Sp}(2g, \mathbb{Z})$. Then

\begin{equation}
\begin{align*}
a) S(g) &\text{ is a bounded subset of } \mathbb{R}. \\
b) f(g) = |S(g)| &\text{ has at least exponential growth}.
\end{align*}
\end{equation}
c) $h(g) = \max \{ m \mid m \in S(g) \}$ has at least exponential growth.

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